CODE : 20677/20895 B.Sc. SEM III EXAMINATION OCT./NOV. 2017

MAT-CC-303 Advanced Calculus-I

TIME : 2:30 HOURS INSTRUCTIONS: (1) All questions are compulsory. TOTAL MARKS:70

(2) Each question carries equal marks.

Q-1 a Find double points of $x^4 + y^4 - 18(x^2 + y^2) + 81 = 0$ and explain their types. 7M

b In which interval the curve $x^2y - x^3 + y = 0$ is concave upward and concave downward and also find its point of inflection. 7M

OR

Q-1 a Prove that function f:R→R is strictly increasing function if $f'(x) > 0\ \forall x \in R$ and hence prove that $f(x) = \frac{\tan x}{x}$ is strictly increasing function in $\left(0, \frac{\pi}{2}\right)$ 7M

b Prove that asymptotes of curve, $x^3 - 4x^2y - xy^2 + 4y^3 - x^2 + 2xy + 3y^2 = 10$ are $x + y = 0$, $2x - 3y - 2 = 0$, $3x - 12y - 1 = 0$ 7M

Q-2 a Evaluate f_x and f_y for $f(x,y) = \frac{(x^{2017}+y^{2017})}{x+y}$ if (x,y)≠(0,0)

$= 0$ if (x,y)=(0,0)

and find value of $xf_x + yf_y$ 7M

b By definition Evaluate : $\lim_{(x,y)\to(2,1)} xy + 4$ if exists. 7M

OR

Q-2 a Find directional derivative of $f(x,y) = \frac{(xy)}{x^2+y^2}$, (x,y)≠(0,0)

$= 0$, (x,y)=(0,0)

at (0,0) along direction of vector $\left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$ 7M

b Discuss continuity of $f(x,y) = \frac{x\cos(x^2+y^2)}{x^2+y^2}$, (x,y)≠(0,0)

$= 0$, (x,y)=(0,0) at (x,y)=(0,0) 7M

Q-3 a If f(x,y) is homogeneous function of x and y of degree m and if its second order partial derivatives exist then $x^2 \frac{\partial^2 f}{\partial x^2} + 2xy \frac{\partial^2 f}{\partial x\, \partial y} + y^2 \frac{\partial^2 f}{\partial y^2} = m(m-1)f(x,y)$ 7M

b If $u = x + y + z, v = x^2 + y^2 + z^2$ and $w = x^3 + y^3 + z^3 - 3xyz \Longrightarrow \frac{\partial(u,v,w)}{\partial(x,y,z)} = 0$ 7M

OR

Q-3 a If $u = \sin\left(\sqrt{x} + \sqrt{y}\right)$ then find $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y}$ 7M

b Prove: f is differentiable homogeneous function of two variable x and y of degree m $\Leftrightarrow xf_x + yf_y = m\, f(x,y)$ 7M

Q-4 a Expand $f(x,y) = \frac{y^2}{x}$ upto second degree in power of (x + 1) and (y − 1) 7M

b Find extreme values of $f(x,y) = x^3 + y^3 - 3x - 12y + 5$ 7M

OR

Q-4 a Expand $e^x \cos y$ in power of x and y up to three degree. 7M

b Find extreme values of $f(x,y) = x^2 + 2y^2 - x$ 7M

Q-5 a State & Prove: Relation between Beta & Gamma Function. 7M

b Find equation of tangent plane and normal line to the surface: $z = \frac{x^2}{2} - \frac{y^3}{3}$ at (2 , 3 , - 1) . 7M

OR

Q-5 a Find equation of tangent line and normal plane of $x^2 - 2y^2 + 3z^2 = 81$, $2x + y - 3z = 8$ at point (2 , 3 , 1) 7M

b State & Prove : Duplication formula. 7M