March – 2015

PAPER NO:M-602
TIME:2:30 HOURS

CODE : 4620
TOTAL MARKS:70

INSTRUCTIONS: (1) All questions are compulsory.
(2) Each question carries equal marks.

| | | | |
|---|---|---|---|
| Q.1 | A | Check which of the following subsets of (R,d) are compact and connected 1.{1,2,3,....,0} 2.$\{\frac{1}{n}$ / n $\in$ N} 3. [0,1] 4. {x$\in$R/ $x^2$+x+1=0} | 08 |
| | B | Prove : Every compact subset of metric space (x ,d) is closed. | 06 |

OR

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|---|---|---|---|
| Q.1 | A | Continuous image of connected subset of metric is connected. | 07 |
| | B | (0,1) is not compact subset of metric space (R,d). | 07 |
| Q.2 | A | Prove : Continuous image of compact subset of metric space is compact. | 07 |
| | B | $E_1$ , $E_2$ $E_3$.........., $E_n$ are connected subspace of metric space ( x ,d) and if $E_i \cap E_j \neq \emptyset$ ,i$\neq$j ,1$\leq$I , j$\leq$n then $\bigcup_{i=1}^{n}$ Ei is connected. | 07 |

OR

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|---|---|---|---|
| Q.2 | A | Prove : Every finite subset of metric space is compact. | 07 |
| | B | Prove : ( x ,d) is metric space and A $\subset$ X is connected. For B $\subset$ X if A $\subset$ B $\subset$ $\bar{A}$ then B is connected. | 07 |
| Q.3 | A | State and prove Dirichlet's test for convergence of improper integrals. | 07 |
| | B | Examine for convergence $\int_1^{\infty} \frac{1}{x\sqrt{x^2+1}} dx$ and $\int_{e^2}^{\infty} \frac{dx}{x\log(\log x)}$. | 07 |

OR

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|---|---|---|---|
| Q.3 | A | State and prove Abel's test for convergence of improper integrals. | 07 |
| | B | State and prove half comparison test for convergence of improper integrals. | 07 |
| Q.4 | A | Prove :The set of rational number is countable. | 07 |
| | B | The set of real number in [0,1] is uncountable. | 07 |

OR

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|---|---|---|---|
| Q.4 | A | State and prove weistrass M –test for uniform convergence of series of functions. | 07 |
| | B | let { $A\alpha/\alpha \in$ I } be indexed collection of subsets of X,then 1.X-U {$A_\alpha$ /$\alpha \in$ I } =$\cap$ {X − $A_\alpha$ /$\alpha \in$ I } 2. X-$\cap$ {$A_\alpha$ /$\alpha \in$ I } =U {X − $A_\alpha$ /$\alpha \in$ I } | 07 |
| Q.5 | A | Cartesian product of two countable set is countable | 07 |
| | B | State and prove Archimedean property of real numbers. | 07 |

OR

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| Q.5 | A | Prove : The set of rational number is not order complete. | 07 |
| | B | If F : A→B is one to one and B is countable then A is countable. | 07 |