B. Sc Semester-VI Mathematics Examination March/April-2016
Mathematical analysis-II
Paper No: M-602

Total Marks:70

Time : 2:30 hours
Paper code: 4626

Instructions:
(1) All questions are compulsory.
(2) Each question carry equal marks.

Que-1. (a) Prove that the usual metric space $(\mathbb{R}, d)$ is not compact. [7]
(b) Let S be a finite subset of the usual metric space $(\mathbb{R}, d)$.
Show that S is connected if and only if S is singleton. [7]

or

Que-1. (a) Let (X, d) be a discrete metric space and $\emptyset \neq A \subset X$.
Show that A is compact if and only if A is finite. [7]
(b) Show that a compact subset of a metric space is closed and bounded.

Que-2. (a) Prove that union of two compact set is compact in any metric space (X, d).
(b) Define *convergence of improper integrals of both kind.*
Determine if the integral $\int\limits_{-\infty}^{0} \frac{1}{\sqrt{3-x}} dx$ is convergent or divergent.
Find its value in case of convergence. [7]

or

Que-2. (a) Show that $\int\limits_{1}^{\infty} \frac{\sin x}{x^p} dx$ converges absolutely if $p > 1$.
(b) State and Prove Abel's test. [7]

Que-3. (a) State and prove comparison test. [7]
(b) Examine the convergence of the improper integral $\int\limits_{0}^{\infty} x^3 e^{-x^2} dx$. [7]

or

Que-3. (a) Let $f : [a, b] \to \mathbb{R}$ be a bounded and integrable function. Show that the function F defined as $F(x) = \int\limits_{a}^{x} f(t)dt$, $a \leq x \leq b$ is continuous on $[a, b]$. [7]
(b) Define *absolute convergence* of an improper integral. Show that $\int\limits_{a}^{\infty} f dx$ exists, if $\int\limits_{a}^{\infty} |f|$ exists, where $a \in \mathbb{R}$. [7]

Que-4. (a) Prove that the set of all rational number is countable. [7]
(b) Let $a, b \in \mathbb{R}$ and $a, b > 0$. Then prove that there exists a positive integer n such that $na > b$. [7]

or

Que-4. (a) Prove that $[0, 1]$ is uncountable. [7]
(b) Prove that every subset of a countable set is countable. [7]

Que-5. (a) State and prove Weierstrass M-test for uniform convergence of series of functions. [7]

(b) Prove that the sequence $\{f_n\}$, where $f_n(x) = \frac{x}{1+nx^2}$, $x \in \mathbb{R}$, converges uniformly on $[a, b]$. [7]

or

Que-5. (a) Test the uniform convergence of the series $\sum_{n=1}^{\infty} \frac{x}{n(1+nx^2)}$ for all $x \in \mathbb{R}$. [7]

(b) In usual notations show that for $x, y \in \mathbb{R}$,

(i) $|x + y| \leq |x| + |y|$,

(ii) $||x| - |y|| \leq |x - y|$. [7]