02/11/NOV - 2017.

B.Sc. Semester VI
Statistics Paper ST 602 Sub code: 4626
(Statistical Inference II)
Total Marks : 70
Duration of Time : $2\frac{1}{2}$ Hours.

Instructions : There are Five compulsory questions in this question-paper All questions carry equal marks. Statistical tables and Graph papers will be provided upon request.

Q 1 (a) Explain the following terms: 06
1. Random Interval
2. Confidence Coefficient
3. Nominal scale

(b) Describe the general method of constructing a 100(1-α) % confidence interval for a parameter Θ. 08

OR

Q 1 (a) What is a Pivotal Quantity in the construction of a confidence Interval? Give the procedure of constructing a 100(1-α)% confidence interval for the difference of means of two normal populations whose variances are unknown but same. 10

(b) Based on an observed random sample of size 16, $x_1, x_2 \ldots\ldots x_{16}$ from a normal population it was computed that $\sum(x_i - \overline{x})^2 = 1560$. Prepare a 90% confidence interval for the population s.d. 04

Q 2 (a) Describe fully the procedure of constructing a 100(1-α) % confidence interval for μ_1- μ_2 based on a random sample of size n from a bivariate normal population BVN($\mu_1, \mu_2, \sigma_1^2, \sigma_2^2, \rho$). 08

(b) Given 3.6, 4.9, 5.2, 3.7, 4.3, 5.4, 4.2, 6.5 are observations of an observed random sample from a normal population N (μ, σ^2), construct a 90% confidence interval for μ. 06

OR

Q2 (a) What are Non-parametric methods? State their advantages over the parametric methods. 09

(b) Examine whether the following sequence of K and M exhibit randomness. Set up hypotheses and test at $\alpha = 0.05$. 05

KKKKKMMKKKKKMMMMKKKKKKKMMMKKKKKKKMK

Q 3 (a) The independent observed random samples of sizes 10 and 13 selected from two normal populations N (μ_1, σ_1^2) and N (μ_2, σ_2^2) gave the sample variances $S_1^2 = 47.2$ and $S_2^2 = 38.0$ respectively. Prepare a 90 % confidence interval for the ratio $\frac{\sigma_1^2}{\sigma_2^2}$. 07

(b) Explain the Wilcoxon signed ranks test for two samples covering (1) purpose (2) nature of Data (3) hypotheses to be tested (4) procedure (5) test statistic (6) Critical Region and (7) its comparison with the sign test. 07

OR

Q 3 (a) Explain Nominal and Ordinal scales of data giving illustrations. 04

(b) Explain Kolmogorov-Smirnov test of goodness of fit in brief. Examine whether the following sample could reasonably be thought to have originated from a Uniform probability distribution over (25,45) applying Kolmogorov-Smirnov test. 10

27.3, 40.1, 34.5, 43.7, 31.5, 31.2, 40.1, 36.0, 33.3, 30.9,
42.0, 34.5, 43.2, 28.5, 30.2

Q 4 (a) Define the following terms: 05

1. Parameter Space
2. Simple Hypothesis
3. Critical Region
4. Significance Level
5. Power Function

(b) Let $X_1 X_2 \ldots\ldots X_{16}$ be a random sample of size 16 from a normal population N (μ, 25) .To test $H_0 : \mu = 50$ against $H_1 : \mu > 50$, it is decided to use a critical region C = $\{ (x_1, x_2 \ldots x_{16}) \mid \bar{x} > 54 \}$. 09

Find the significance level and the power function of the test characterized by the critical region C. Find the probabilities of type II errors at $\mu = 52$ and 54.

OR

Q 4 (a) Distinguish between
1. Simple Hypothesis and Composite Hypothesis — 06
2. Type I Error and Type II Error
3. Significance level and the Power of the test

(b) Given $X_1, X_2 \ldots X_6$ is a random sample of size 6 from a Bernoulli Probability Distribution with the probability function — 08

$$f(x,\theta) = \theta^x (1-\theta)^{1-x} \quad x=0,1 \quad \theta \in \{\tfrac{1}{2}, \tfrac{2}{5}\}$$
$$= 0 \quad \text{Elsewhere.}$$

Find the probabilities of Type I Error and Type II Error of the CR $C=\{(x_1,x_2,\ldots\ldots x_6) \mid \sum x_i \le 2\}$ is used to test $H_0: \theta = \frac{1}{2}$ against $H_1: \theta = \frac{2}{5}$.

Q 5 (a) Define a best critical region of significance level α to test a Simple hypothesis against a Simple alternative hypothesis. — 03

(b) State and prove the Neyman-Pearson theorem to find a best critical region to test a simple Null hypothesis against a simple alternative hypothesis. — 11

OR

Q 5 (a) Obtain a most powerful test of size α to test $H_0: \theta = \theta_0$ $H_1: \theta = \theta_1$ based on a random sample of size n from a Poisson Probability distribution with mean θ. $(\theta_1 > \theta_0)$ — 07

(b) What is a UMP test of size α ? Find a UMP size 0.01 test to test $H_0: \theta = 3$ against $H_1: \theta < 3$ based on a random sample of size 9 from an exponential probability distribution having mean θ. What is its power function? — 07