M.Sc (Sem. I) Examination

Paper – III : Inference – I,

Time :2 Hours] Dec - 2016 : Code - 2754 **[Total Marks :70**

Instruction : **(I) : All Questions are compulsory**

(II) : All Questions carry equal marks.

1. (a) Defien MVUE. State and prove Rao –Blackwell theorem.

(b) Define unbiased estimator. Obtain two unbiased estimators for σ in case of normal $N(\mu, \sigma^2)$ distribution where μ is known. Which estimator do you prefer? Why?

OR

1. (a) Define consistent estimator of θ. Show that If T is consistent for θ then g(T) will also be consistent for $g(\theta)$ but such property is not true in case of unbiased estimator. Verify it by example

(b) Obtain MVUE for θ using Rao –Blackwell theorem based on a random sample from uniform distribution $U(0, \theta)$.

2. (a) Define complete family of distribution. Show that one parameter exponential family is a complete family.

(b) State UMVUE.. Obtain UMVUE for $e^{-\lambda}$ based on a random sample from Poisson distribution with mean λ, $\lambda > 0$.

OR

2. (a) State and prove Cramer –Rao inequality. Obtain CRLB for an unbiased estimator for $e^{-\lambda}$ based on a single observation from Poisson distribution with mean λ, $\lambda > 0$. Show that UMVUE of $e^{-\lambda}$ does not attained the CRLB.

(b) State and prove Neyman Fisher Factorization theorem.

3. (a) Prove that If MVUE exists, it is unique.

(b) Define MLE. Show that MLEs are asymptotical normally distributed. Let $X_1, X_2, \ldots, X_n$ be a random sample from the exponential distribution with location parameter μ and scale parameter θ. Obtain MLEs of the parameters.

OR

3. (a) Define method of moments for estimating the parameters of the given distribution. Obtain moment estimators of the parameters of $U(a, b), a<x<b$ distribution based on a random sample of size n.

(b) Let $(X_1, X_2, \ldots, X_n)$ be i.i.d with. $f(x,\theta) = \theta e^{-x\theta}$, $\theta > 0$
Obtain MLE of θ. Find its asymptotic S.E.

4. (a) Discus the situations with illustration where the parameter of the distribution is it self a random variable. In which estimation method such concept is used? How does it differ from classical method of estimation?

(b) Define Bayes risk and Bayes estimator. Obtain general form of Bayes estimator under squared error loss functions.

OR

4. (a) A random sample of size n is taken from binomial distribution with mean $n\theta$. If prior distribution of θ is beta distribution with parameters a and b, $a < b$ obtain Bayes estimator of θ under squared error loss function.

(b)A random sample of size n is taken from Poison distribution with parameter $\lambda, > 0$. If λ, has prior p.d.f. Gamma(α, β), α , $\beta > 0$ then obtain Bayes estimator of λ, under squared error loss function .

5. (a) Let X_1 and X_2 is from $N(\theta,1)$ then obtain information contained in given random sample.

(b) Construct 95% confidence interval for σ, in case of $N(\mu, \sigma^2)$ distribution, when μ is known based on a random sample of size n.

OR

5. (a) Discuss shortest length confidence interval. How do you construct it? Explain by example.

(b) Let $(X_1, X_2, \ldots, X_n)$ be i.i.d with $f(x,\theta) = \theta e^{-\theta x}$, $x > 0$, $\theta > 0$. Obtain (1-α)100% confidence interval for θ.