M. Sc. (Mathematics) Semester-I
Paper 4: Number Theory
MARCH - APRIL - 2016
Code: 2751

Total Marks: 70

Q-1. (a) Prove that $32 \mid (a^2+3)(a^2+7)$, for any odd integer a. [7]

(b) Given integers a and b, not both of which are zero, show that [7]
$\{ax+by / x, y \text{ are integers}\} = \{kd / k \text{ is integer}\}$, where
$d = \gcd(a,b)$.

OR

Q-1. (a) Prove: the qube of any integer has one of the forms $9k$, $9k+1$ or $9k+8$. [5]

(b) Discuss: Euclidean Algorithm. [9]

Q-2. (a) Find integers x and y satisfying $\gcd(138,\ 42) = 138x + 42y$. [7]

(b) For positive integers a and b, prove that $\gcd(a,b)lcm(a,b) = ab$. [7]

OR

Q-2. (a) State and prove Fundamental Theorem of Arithmetic. [9]

(b) Prove: $\sqrt{2}$ is an irrational number. [5]

Q-3. (a) If $ca \equiv cb \pmod{n}$, then prove that $a \equiv b \ (\text{mod} \frac{n}{d})$, where $d = \gcd(c,n)$. [7]

(b) Find the remainder when the sum $1^7 + 2^7 + 3^7 + ... + 100^7$ is divided by 4. [7]

OR

Q-3. (a) Solve the linear congruence: [6]
$$6x \equiv 15 \pmod{21}.$$

(b) State and prove Chinese Remainder Theorem. [8]

PTO

Q-4. (a) Find the unit digit of 7^{100}. [7]

(b) If $n > 1$, $(n-2)! \equiv 1 (\bmod n)$, then prove that n is prime. [7]

OR

Q-4. (a) Find the remainder when 9^{100} is divided by 21. [5]

(b) Prove: the quadratic congruence $x^2 + 1 \equiv 0 (\bmod p)$, where p is an odd prime has a solution if and only if $p \equiv 1 \ (\bmod 4)$. [9]

Q-5. (a) Find the formula of $\sigma(n)$ for $n > 1$. [7]

(b) Prove that the Euler function φ is multiplicative. [7]

OR

Q-5. (a) Prove that the Mobius function μ is multiplicative. [7]

(b) Prove that $\varphi(n)$ is an even integer, for all $n > 2$. [7]

—X— X—