November - 2015

M. Sc. (Mathematics) Sem. - I

Paper 4: Number Theory

code: 2751

Total Marks: 70

Q-1 (a) Prove that the qube of any integer has one of the forms $9k$, $9k+1$ or $9k+8$. [8]

(b) Determine Euclidean Algorithm. [6]

OR

Q-1 (a) Given integers a and b, not both of which are zero, show that $\{ax+by / x, y \text{ are integers}\} = \{kd / k \text{ is integer}\}$, where $d = \gcd(a,b)$. [7]

(b) If gcd(a, b)=1, then show that gcd(a^3, b^3)=1. [7]

Q-2 (a) State and prove Fundamental Theorem of Arithmetic. [8]

(b) If p_n is the n^{th} prime number then show that $p_n \le 2^{2^{n-1}}$. [6]

OR

Q-2 (a) Prove that there are an infinite number of primes. [7]

(b) Prove: A composite integer n will always possess a prime divisor p satisfying $p \le \sqrt{n}$. [7]

Q-3 (a) If $ca \equiv cb \pmod{n}$, then prove that $a \equiv b \ (\text{mod}\frac{n}{d})$, where $d = \gcd(c,n)$. [7]

(b) Find the remainder when the sum $1^7 + 2^7 + 3^7 + ... + 100^7$ is divided by 3. [7]

OR

Q-3 (a) Solve the linear congruence: [7]

$$6x \equiv 15 \pmod{21}.$$

(b) If p is a prime then prove that $a^p \equiv a \pmod{p}$, for any integer a. [7]

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Q-4 (a) State and prove Chinese Remainder Theorem. [7]

(b) If n is prime then, prove that $(n-2)! \equiv 1 (\bmod n)$. [7]

OR

Q-4 (a) Prove: the quadratic congruence $x^2 + 1 \equiv 0 (\bmod p)$, where p is an odd prime has a solution if and only if $p \equiv 1 \ (\bmod 4)$. [7]

(b) If p and q are distinct primes such that $a^p \equiv a \ (\bmod q)$ and $a^q \equiv a \ (\bmod p)$, then show that $a^{pq} \equiv a \ (\bmod pq)$. [7]

Q-5 (a) Find the formula for $\tau(n)$ for $n > 1$. [7]

(b) For $n > 2$, prove that $\varphi(n)$ is an even integer. [7]

OR

Q-5 (a) Find the formula for $\sigma(n)$ for $n > 1$. [7]

(b) Prove that the Mobius function μ is multiplicative. [7]

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