M.Sc. (Mathematics) Semester – 1, MARCH 2017

Paper – 1: Real Analysis (Code: 2748)

Time: 2 ½ Hours

Total Marks: 70

Q.1 (a) Let $X \neq \emptyset$ and $\mathcal{C} \subseteq X$. Prove that there exist a smallest σ –algebra of sets containing $\mathcal{C}$. [7]

(b) Let $a \in \mathbb{R}$. Show that the interval (a, ∞) is measurable. [7]

OR

Q.1 (a) Show the existence of a non-measurable set. [7]

(b) Prove that a Lebesgue measure function m is countable subadditive and then show that it is countable additive also. [7]

Q.2 (a) If f is a measurable function and $f = g$ $a.e.$ then prove that g is measurable function. [7]

(b) Let D be a Borel set, α be any real number and $f: D \to [-\infty, \infty]$. Prove that the set $\{x \in D : f(x) < \alpha\}$ is Borel set if and only if the set $\{x \in D : f(x) \geq \alpha\}$ is Borel set. [7]

OR

Q.2 (a) Let $A, B \subseteq \mathbb{R}$. Show in usual notations: [7]

(i) $\mathcal{X}_{A \cap B} = \mathcal{X}_A \mathcal{X}_B$ (ii) $\mathcal{X}_{A \cup B} = \mathcal{X}_A + \mathcal{X}_B - \mathcal{X}_A \mathcal{X}_B$

(b) Let $\{f_1, f_2, \dots \dots \dots\}$ be a countable collection of measurable functions with same domain then prove that $\overline{lim} f_n$ and $\underline{lim} f_n$ are measurable function. [7]

Q.3 (a) Let $f: [-1, 6] \to \mathbb{R}$ be given by $f(x) = 2x^2$. Find upper sum and lower sum of f for a subdivision $-1 < \frac{-1}{2} < 0 < 1 < 2 < 5 < 6$ of : $[-1, 6]$. [7]

(b) Let f be a bounded measurable function defined on a set E of finite measure and $a \in \mathbb{R}$ then prove that $\int_E af = a \int_E f$ [7]

OR

Q.3 (a) Give an example of a function which is not Riemann integrable. Justify your answer. [7]

(b) Let $\varphi: \mathbb{R} \to [-\infty, \infty]$ be defined as $\varphi(x) = \begin{cases} -2 & if\ x \in (1,2] \\ 2 & if\ x \in (3,5) \\ 0 & otherwise \end{cases}$. [7]

Find (i) $\int \varphi$ (ii) $\int_{(-3,2]} \varphi$ (iii) $\int_{(1.5,3.5)} \varphi$

Q.4 (a) Let f and g be two non-negative measurable functions. If f is integrable over a set E and $f(x) > g(x), x \in E$ then prove that the function g is integrable over E and $\int_E (f-g) = \int_E f - \int_E g$ [7]

(b) Let f be a real valued function and $c \in \mathbb{R}$. Prove: [7]

(i) $(-f)^+ = f^-$ (ii) $(-f)^- = f^+$

(iii) $(cf)^+ = \begin{cases} cf^+ & if\ c \geq 0 \\ -cf^- & if\ c < 0 \end{cases}$ (iv) $(cf)^- = \begin{cases} cf^- & if\ c \geq 0 \\ -cf^+ & if\ c < 0 \end{cases}$

OR

Q.4 (a) Let f and g be non-negative measurable functions defined on the same set E of finite measure. Prove that $\int_E (f+g) = \int_E f + \int_E g$ [9]

(b) Let $f: \mathbb{R} \to \mathbb{R}$ be defined as $f(x) = -x^2 + 3x - 2$. [5]

Find $f^+(-3), f^-(3), f^+(-1), f^{-1}, f^-(0)$.

Q.5 (a) State and prove Riemann Lebesgue Theorem. [7]

(b) Let f be integrable function over $[a, b]$ and $F(x) = \int_a^x f(t)dt$ then show that F is a continuous function of bounded variation on [a, b]. [7]

OR

Q.5 (a) Define absolutely continuous function and show that every absolutely continuous function is continuous function. [7]

(b) Let $f : [1,5] \to \mathbb{R}$ be defined as $f(x) = x^2 + 1, \forall x \in [1,5]$. Find p, n and t for a subdivision $1 < 2 < 3 < 4 < 5$. [7]
