March-2017

M.Sc. (Mathematics) Semester – 1

Paper – 2: Topology-I (Code: 2749)

Time: 2 ½ Hours **Total Marks: 70**

Q.1 (a) Define topological space. Discuss with all detail co-finite topology. [7]

(b) In a topological space prove that an arbitrary union of open sets is open. [7]

OR

Q.1 (a) In a topological space prove that a finite intersection of open set is open. [7]

(b) Let $\tau = \{\emptyset, X, \{b\}, \{a,b\}, \{a,b,d\}\}$ be a topology on $X = \{a,b,c,d\}$. Find neighborhood of (i) a (ii) b (iii) c. [7]

Q.2 (a) Define following terms: [7]

(i) Limit point (ii) Adherent point (iii) Isolated point

And in usual notations prove that $A \subset B \Longrightarrow \bar{A} \cup \bar{B}$.

(b) In usual notations prove: (i) $\overline{A \cup B} = \bar{A} \cup \bar{B}$ (ii) $\overline{A \cap B} \subset \bar{A} \cap \bar{B}$ [7]

OR

Q.2 (a) Let $\tau = \{\emptyset, X, \{a\}, \{b\}, \{a,b\}, \{c,d\}, \{a,c,d\}, \{b,c,d\}\}$ be a topology for $X = \{a,b,c,d\}$. Then prove that the collection $B = \{\{a\}, \{b\}, \{c,d\}\}$ is a base for τ. [7]

(b) Prove that the function $f: (X, \tau) \to (Y, u)$ is continuous if and only if $f^{-1}(V)$ is open in X for every open set V in Y. [7]

Q.3 (a) Show that every Hausdorff space is a T_1 space. Is the converse true? [7]

(b) Prove that a normal space is a regular space. [7]

OR

Q.3 (a) In usual notations prove: $\bar{A} = A \cup A'$. [7]

(b) In usual notations prove: π_1 and π_2 are continuous functions. [7]

Q.4 (a) Prove that limit of a sequence in Hausdorff space is unique. [7]

(b) Prove that every metric space is a T_2 space. [7]

OR

Q.4 (a) State and prove Uniform Limit Theorem. [7]

(b) State and prove Pasting Lemma. [7]

Q.5 (a) Prove that A is nowhere dense set if and only if $X - \bar{A}$ is dense set. [7]

(b) Prove that Z is a nowhere dense set. [7]

OR

Q.5 (a) Let (X, d) be a complete metric space then prove that every closed subspace of X is complete. [7]

(b) Prove that contraction on X is a continuous function. [7]
