M.Sc. Physics Examination

Semester – I

Paper No – C103 Mathematical Methods in Physics

Paper Code – 4514

Time: 2 Hours 30 min   Dec - 2016   Maximum Marks 70

Notes: (1) - All questions are compulsory. (2) Number in square bracket indicate marks

Que–1:

(a) Calculate the Laplacian of $1/r$. [3]

(b) If $\delta(t)$ is Dirac delta function in 't', what is the value of $\int_0^\infty \delta(t)\exp(-jwt^2)dt$. [4]

(c) If $\bar{L}=\vec{r}\, X\, \vec{p}=-i\hbar\vec{\nabla}$ then show that $\bar{L}\, X\, \bar{L}=i\hbar\bar{L}$ [7]

OR

(a) A vector force field is given by $\vec{F}=-y\hat{i}+z\hat{j}+x^2\hat{k}$. Find the curl of force field and then convert the result into cylindrical coordinates. [6]

(b) Express the Cartesian component of dell $(\vec{\nabla})$ operator in cylindrical coordinates. [8]

Que–2:

(a) Evalute $\int_0^\infty \frac{x^2dx}{(x^2+4)(x^2+9)}$ using the residue theorem. [8]

(b) Prove that $u=e^{-x}(x\sin y - y\cos y)$ is harmonic. [6]

OR

(a) Find the value of the integral

$\int_C \frac{z^3dz}{z^2-5z+6}$ Where C is a closed contour defined by the equation $2|z|-5=0$ [8]

(b) Find residue of function $\frac{z-2}{z^2(z-1)^2}$ at z=0 and z=1. [6]

Que–3:

(a) Obtain the generating function for Bessel's Polynomial. [7]

(b) Solve the differential equation $\frac{d^2y}{dx^2}-\frac{1}{x}\frac{dy}{dx}+4x^2y=x^4$, by changing the independent variable method. [4]

(c) Prove that $J_{-n}(x)=(-1)^n J_n(x)$. [3]

OR

(a) Write and plot Bessel's polynomials $J_0(x)$ and $J_1(x)$. Also indicate root. [7]

(b) Prove the recurrence formula for Bessel' function

$$xJ'_n(x)=nJ_n(x)-xJ_{n+1}(x)$$ [4]

(c) solve given differential equation: $x\frac{dy}{dx}-y=x^2$ at y(1)=1. [3]

Que–4:

(a) Deduce the Rodrigue's formula $P_n(x)=\frac{1}{2^n n!}\frac{d^n}{dx^n}(x^2-1)^n$. [7]

(b) Find the Laplace transform of t^3e^{-3t} . [4]

(c) Write generating functions for Legendre and Lagguerre's polynomial. [3]

OR

(a) Write and plot Legendre's polynomials $P_0(x)$, $P_1(x)$ and $P_2(x)$. Also indicate root. [7]

(b) Find the Laplace transform of $\sin\sqrt{t}$. [4]

(c) Prove that $H_2(x)=4x^2-2$ and $L_2(x)=x^2-4x+2$. [3]

Que–5:

(a) Draw the graph of the periodic function and find its Laplace transform.

$$f(x)=\begin{cases} t & 0<t<\pi \\ \pi-t & \pi<t<2\pi \end{cases}$$ [6]

(b) Find the Fourier Transform of e^{-at} . [2]

(c) A wave is represented as $f(x)=\begin{cases} -1 & -\pi<x<0 \\ 1 & 0<x<\pi \end{cases}$ which is periodic over 2π. Express this function as a series of sine and cosine function. [6]

OR

(a) Using Laplace transforms, find the solution of the initial value problem (IVP) $y''-4y'+4y=64\sin 2t$; $y(0)=0$, $y'(0)=1$. [6]

(b) Find the Laplace transform of $\frac{\sin 2t}{t}$. [2]

(c) Derive (i) Time shifting and (ii) Frequency shifting property of Fourier transform. [6]