M.Sc. Physics Examination

Semester – I NOV. 2014

Paper No – C103 Mathematical Methods in Physics

Paper Code – 4514

Time: 2 Hours 30 min Maximum Marks 70

Notes: (1) - All questions are compulsory. (2) Number in square bracket indicate marks

Q.1 (a) Obtain the scale factors for curvilinear coordinate system. [6]

(b) Find $\nabla^2(r^2)$ *and* $\nabla^2(1/r^2)$ using spherical polar coordinate system. [4]

(c) Prove that curl of electric field is zero. [4]

OR

(a) Express the Cartesian component of dell (∇) operator in spherical coordinates. [8]

(b) Calculate divergence and curl of position vector. [6]

Q. 2 (a) State and prove Cauchy's integral Theorom. [7]

(b) Evaluate the following integral using residue theorem [7]

$$\int_0^{2\pi} \frac{d\theta}{5 - 4\cos\theta}$$

OR

(a) Find the residues of the following functions at given points [9]

$$(i)\ \cot z \text{ at } z = 0 \quad (ii)\ \frac{z}{(3z-1)(5-z)} \text{ at } z = -\frac{1}{3}, 5$$

(b)Find whether the functions given below are analytic or not. [5]

(i) z^3 (ii) $\sin z$(iii) z^*

Q. 3 (a) Prove following recursion relation for Legendre Polynomial [7]

$$J_{n-1}(x) + J_{n+1}(x) = \frac{2n}{x} J_n(x).$$

(b)Obtain the generating function for Bessel's polynomials. [7]

OR

(a) Prove that [5]

$$\int_{-1}^{+1} (x^2 - 1)\, P_{n+1} P'_n dx = \frac{2n(n+1)}{(2n+1)(2n+3)}$$

(b) Solve $\frac{d^2y}{dx^2} + 4\frac{dy}{dx} + 5y = 0$, if y = 2 and $\frac{d^2y}{dx^2} = \frac{dy}{dx}$, when x = 0 [5]

(c) Using Rodrigue's formula, prove that [4]

$$\int_{-1}^{+1} P_n(x)\,dx = 0, (n \neq 0)$$

Q. 4 (a) Solve the given simultaneous differential equations for $x(t)$ and $y(t)$ with the help of Laplace transform methods

$$\frac{dx}{dt} + x + 4y = 10 \qquad \&x - \frac{dy}{dt} - y = 0$$

Given that $x(0) = 4, y(0) = 3$ [10]

(b) Write formula for generating function of Hermite and Laguerre functions. [4]

OR

(a) Using the method of separation of variables obtain the solution of a wave equation applicable to the spherical membrane. [8]

(b) Show that the value of the integral [6]

$$\int_{-1}^{+1} x^2 P_3(x)\,dx = 0$$

Q.5 (a) Find $f(t)$ if $L\{f(t)\} = \log\left(\frac{s+2}{s-3}\right)$ [7]

(b) Solve the given equation for the response of $i(t)$ [7]

$$\frac{di}{dt} + 2i + 5\int_0^t i dt = u(t)$$

Given that $i(0) = 0$

OR

(a) If Fourier transform of Dirac Delta function $\delta(t)$ is 1, then prove that [7]

$$\delta(t) = \frac{1}{\pi}\int_0^{\infty} \cos \alpha t\, d\alpha$$

(b) Using the Laplace transform of Dirac delta function $\delta(t)$, Solve the following initial value problem (IVP)

$$y'' + 3y' - 10y = 4\delta\,(t-2)$$

Boundary conditions are given as $y(0) = 2,\ y'^{(0)} = -3$ [7]