Nov.-2017

M.Sc (Sem. I) Examination (Statistics)

Paper – 3 : Inference – I (Theory of Estimation)

Code - 2754

Time : 2.30 Hours] [Total Marks :70

Instruction : (I) : All Questions are compulsory

(II) : All Questions carry equal marks.

1. (a) If X_1 and X_2 follows Poisson (θ) then prove that,
   i. $T_1 = X_1 + X_2$ are sufficient statistics for θ.
   ii. $T_2 = X_1 + 2X_2$ is not sufficient for θ.

(b)) Define complete statistic. Show that if sufficient statistic is complete, it is always minimal sufficient.

OR

1.(a)Define Fisher information contained in a statistic T. State and prove Lehman-Scheffe theorem.

(b) Obtain UMVUE for P(X=r) , based on a random sample from a exponential variate with parameter $\theta>0$.

2. (a)) Let $x_1, x_2,\ldots, x_n$ be a random sample from the exponential distribution with location parameter μ and scale parameter σ. Obtain MLE of $\mu+2\sigma$.

(b) State and prove Cramer –Rao (C-R)inequality. Obtain CRLB for an unbiased estimator for p based on a random sample of size n from Bernoulli distribution with parameter p, $0 < p < 1$.

OR

2. (a) Define complete family of distribution. Show that exponential family is a complete family.

(b) Let X_i Follows $N(\theta,1)$ and statistic $T_1 = lX_1 + mX_2$, then derive information contained in statistic T_1 when (i) $l = m$.

3. (a) Define MVUE. State and prove Rao –Blackwell theorem.
(b) Define MLE. State and prove invariance property of MLE.

OR

3. (a) Let X1,X2,…..,Xn be a random sample form the p.d.f. f(x) = 1/b-a ; a <x<b, obtain the MLE's of a and b. Also obtain moment estimator of a & b.

. (b) State UMVUE. Show that it is unique, if it exist. Obtain UMVUE for $e^{-\lambda}$ based on a random sample from Poisson distribution with mean λ, λ > 0.

4. (a) Define loss function, prior distribution and posterior distribution, with illustrations

(b) Explain the Method of Minimum Chi-square.

OR

4. (a) Let $(X_1, X_2, ..., X_n)$ be i.i.d with $f(x,\theta) = \theta e^{-\theta x}$, $x > 0$, $\theta > 0$. Obtain (1-α)100% confidence interval for θ.

(b) A random sample of size n is taken from Poison distribution with parameter λ, λ > . If λ has gamma gamma(a,b), a>0, b> 0prior distribution then obtain Bayes estimator of λ under squared error loss function .

5. (a) State confidence interval for g(θ) and interpret it. Discuss the pivotal method of construction of confidence interval with illustration.

(b) Construct 95% confidence interval for θ, in case of U(0, θ) uniform distribution based on a largest order statistic of a random sample of size n.

OR

5. (a) A random sample of size n is taken from binomial distribution with mean np. If prior distribution of p is beta type – I distribution with parameters a and b, a < b obtain Bayes estimator of p under squared error loss function.

(b) Let $X_1, X_2, \ldots, X_n$ be a random sample of size from N (θ, σ^2) Obtain 95% confidence interval for σ^2.