Dec- 2016 :

M.Sc.(Sem. –I) EXAMINATION

STATISTICS :PARER 01 - 2752

Linear Algebra

TIME : Two Hours. TOTAL MARKS:70

Note: (i) All Questions are Compulsory

(ii) All Full Questions carry equal marks.

1\. (a) If A : m× m , B: m × n , C: n× m and D: n× n are matrices and P: (m+ n) × (m+ n) is a nonsingular matrix such that **8**

$$P = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \text{ then show that}$$

$$P^{-1} = \begin{bmatrix} A^{-1} + A^{-1}BQCA^{-1} & -A^{-1}BQ \\ -QCA^{-1} & Q \end{bmatrix} \text{ if A is}$$

nonsingular and $Q = (D - CA^{-1}B)^{-1}$.

(b) Explain (i)linearly independent (ii)linearly dependent (iii) basis (iv) orthogonal vector **6**

OR

1\. (a) Show that square matrix A is non-singular iff all its columns are linearly independent **8**

(b) For any two matrices $A_{p\times m}$ and $B_{p\times n}$ prove that **6**

Max[ρ(A),ρ(B)]≤ rank(A,B) ≤rankA + rankB

2\. (a) (1)Show that {(1,1,0),(1,0,1),(0,1,1)} is a basis of V_3(R).
(2) prove that A set of orthogonal vectors is always L.I. **8**

(b) Show that the system of linear non-homogeneous equation $A\underline{x} = \underline{b}$ is consistent, if ρ (A, b)= ρ (A). **6**

OR

2. (a) Show that for matrix $A_{n\times n}$ Rank(A)+ Rank(I-A) –n= Rank((I-A) A) 8

(b) For any two matrices A and $\boldsymbol{B}$ prove that 6

$\boldsymbol{\rho}$ (AB)= $\boldsymbol{\rho}$ (BA)=. $\boldsymbol{\rho}$ (A) , B is non-singular.

3. (a) Show that matrix $\boldsymbol{A}_{m\times n}$ is idempotent matrix iff Rank(A)+ Rank(I-A) =n 6

(b) Show that $\overline{\boldsymbol{A}}$ exists iff H=$\overline{A}\,\boldsymbol{A}$ is idempotent 8

OR

3 (a) Explain types of g-inverse 6

(b) Reduce the symmetric matrix 8

$$\boldsymbol{A}=\begin{bmatrix} \mathbf{4} & \mathbf{2} & \mathbf{1} \\ \mathbf{3} & \mathbf{1} & \mathbf{0} \\ \mathbf{1} & \mathbf{0} & \mathbf{-1} \end{bmatrix}$$ to a diagonal matrix D.

4. (a) 6

Let $\bar{A}$ be any g-inverse of A and H=$\overline{A}\,A$, then prove that general solution of a consistent non-homogeneous equation Ax=y is x=$\overline{A}\,y + (i - H)z, z\ is\ any$ arbitrary vector.

(b) Define (1) a g- inverse A^{-} of a matrix A and 8

(2) The Moore-Penrose g-inverse of a matrix.

Further find a g-inverse of $\begin{bmatrix} 3 & -1 & 1 \\ 2 & 3 & -3 \end{bmatrix}$.

OR

4. (a) Write a note on(i) Types of Quadratic form 8

(b) Show that $\overline{\boldsymbol{A}}$ exists iff A $\overline{\boldsymbol{A}}\,\boldsymbol{A} = \boldsymbol{A}$ 6

5 (a) Explain Gram Smith orthogonalization process 6

(b) Explain Sylvester's Criterion for positive definite form. 8

Is the following quadratic form is positive definite?

i. $Q(x) = X_1{}^2 + 2X_2{}^2 + 7X_3{}^2 - 2X_1X_2 + 4X_1X_3 - 6X_2X_3$

OR

5 (a) Prove that Moore-Penrose g-inverse of a matrix is unique. 8

(b) Define following terms 6

(i) equivalence of Quadratic forms

(ii) Quadratic forms (iii)Rank of Quadratic form

(iv Normal form of Quadratic form

(v) Signature of Quadratic form (vi)Diagonal form of a Quadratic form.