oct-NOV- 2017

M.Sc.(Sem. –I) EXAMINATION

STATISTICS :PARER 01 , SUB CODE- 2752

Linear Algebra

TIME : Two Hou2.30hrs. TOTAL MARKS:70

Note: (i) All Questions are Compulsory

(ii) All Full Questions carry equal marks.

1. (a) Let $\bar{A}$ be any g-inverse of A and H=$\bar{A}\,A$, then prove that general solution of a consistent non-homogeneous equation Ax=y is x=$\bar{A}\ y + (i - H)z, z\ is\ any$ arbitrary vector. **8**

   (b) Explain (i)linearly independent (ii)linearly dependent (iii) basis (iv) orthogonal vector **6**

OR

1. (a) Show that the system of linear non-homogeneous equation $\boldsymbol{A}\underline{\boldsymbol{x}} = \underline{\boldsymbol{b}}$ is consistent, if $\boldsymbol{\rho}$ (A, b)= $\boldsymbol{\rho}$ (A). **6**

   (b) Attempt the following: **8**

   (1) Determine whether the set of vectors
   { (1,0, -1) , (1, 2, 1) , (0, -3, 2)} form a basis of $R^3$.
   5)}.
   (2)A set of orthogonal vectors is always L.I.

2. (a) If A : m× m , B: m × n , C: n× m and D: n× n are matrices and P: (m+ n) × (m+ n) is a nonsingular matrix such that **8**

$$P = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \text{ then show that}$$

$$P^{-1} = \begin{bmatrix} A^{-1} + A^{-1}BQCA^{-1} & -A^{-1}BQ \\ -QCA^{-1} & Q \end{bmatrix} \text{ if A is}$$

nonsingular and $Q = (D - CA^{-1}B)^{-1}$.

(b) 6

Prove that (in usual notations)
$\rho(AB) \leq \min(\rho(A), \rho(B))$.

OR

2. (a) Show that square matrix A is non-singular iff all its rows are linearly independent. 8

(b) For any two matrices $\boldsymbol{A}$ and $\boldsymbol{B}$ prove that 6

$\boldsymbol{\rho}$ (AB)= $\boldsymbol{\rho}$ (BA)=. $\boldsymbol{\rho}$ (A) , B is non-singular.

3. (a) Show that matrix $\boldsymbol{A_{m\times n}}$ is idempotent matrix iff Rank(A)+ Rank(I-A) =n 6

(b) Show that $\overline{\boldsymbol{A}}$ exists iff A $\overline{\boldsymbol{A}}\,\boldsymbol{A} = \boldsymbol{A}$ 8

OR

3 (a) Show that $\overline{\boldsymbol{A}}$ exists iff H=$\overline{\boldsymbol{A}}\,\boldsymbol{A}$ is idempotent 6

(b) Define (1) a g- inverse $A^-$ of a matrix A and 8

(2) The Moore-Penrose g-inverse of a matrix.

Further find a g-inverse of $\begin{bmatrix} 3 & -1 & 1 \\ 2 & 3 & -3 \end{bmatrix}$.

4. (a) Define Moore-Penrose g-inverse of a matrix. Prove that it is unique. 8

(b) Explain Sylvester's Criterion for positive definite form. 6

Is the following quadratic form is positive definite?

i. $Q(x) = X_1^{\,2} + 2X_2^{\,2} + 7X_3^{\,2} - 2X_1X_2 + 4X_1X_3 - 6X_2X_3$

OR

4. (a) Write a note on(i) Types of Quadratic form 8

(b) Explain Gram Smith orthogonalization process 6

5 (a) Explain type of g-inverse 8

(b) Reduce the quadratic form **6**

$Q=\mathbf{10x_1^2 + x_2^2 + x_3^2 - 6x_1x_2}$ to its diagonal forms

and determine its types. (06)

OR

5 (a) Prove that any given quadratic form can be transform to its normal form. **6**

(b) Find a canonical expression of a given quadratic form Q(X). **8**

i. $Q(x) = X_1^2 - 2X_2^2 - 27X_3^2 + 2X_1X_2 - 4X_1X_3 + 14X_2X_3$