Code: 2938

M.Sc. (IT) Semester – 2

Paper – 6 : Linear Algebra

April 2017.

Time: 2 ½ Hours | Total Marks: 70

Q.1 (a) Define (i) Vector space (ii) Vector subspace [7]

(b) Let V be a vector space over R. For $x \in V$, prove that [7]

(i) $0 \cdot x = \theta$

(ii) $(-1) \cdot x = -x$.

OR

Q.1 (a) Show that a set $S = \{(x,y,z) \in R^3 | \, x + 2y + z = 0\}$ is a vector subspace of R^3. [7]

(b) If W_1 and W_2 are subspaces of a vector space V then show that $W_1 \cap W_2$ is also a subspace of V. [7]

Q.2 (a) Let $T: R^2 \to R^3$ be defined as $T(x,y) = (2x, 3y, x+y),\ \forall\, (x,y) \in R^2$. Check whether T is a linear transformation or not. [7]

(b) Let V and W be two vector spaces and $T: V \to W$ be a linear map. Show that $kerT$ is a subspace of V. [7]

OR

Q.2 (a) Verify rank-nullity theorem for a linear map $T: R^2 \to R^2$ defined as [7]

$T(x,y) = (x-y, x+y),\ \forall\, (x,y) \in R^2$.

(b) Find a matrix with respect to standard basis and associated to a linear transformation $T: R^3 \to R^3$ given by [7]

$T(x,y,z) = (x+y+z, -x-y-z, x-y+z),\ \forall\, (x,y,z) \in R^3$.

Q.3 Give graphical and matrix representation of the following special linear transformations: [14]

(i) Identity (ii) Shear (iii) Rotation (iv) Stretching along Y–axis.

OR

Q.3 (a) Find rank of the following matrices: [7]

$$(i) \begin{bmatrix} 1 & 0 & 2 & 1 \\ 0 & 2 & 4 & 2 \\ 0 & 2 & 2 & 1 \end{bmatrix} \qquad (ii) \begin{bmatrix} 1 & 2 & 1 \\ 9 & 5 & 2 \\ 7 & 1 & 0 \end{bmatrix}$$

(b) Give graphical and matrix representation of the following special linear transformations: [7]

(i) Stretching along X–axis (ii) Projection

Q.4 (a) Explain: (i) Norm (ii) Dot product. [7]

(b) Let V be a vector space. Define $d: V \times V \to R$ as $d(x,y) = |x - y|$. Show that d is a metric on V. [7]

OR

Q.4 (a) Let V be an inner product space. For $x, y \in V$ prove that: [7]

(i) $|\|x\| - \|y\|| \leq \|x - y\|$

(ii) $4\langle x, y\rangle = \|x + y\|^2 - \|x - y\|^2$

(b) (i) Find norm of a vector $v = (1, -2, 6, 3)$ [7]

(ii) Find angle between vectors $x = (3, 4)$ and $y = (-8, 6)$

(iii) Define orthogonal vectors.

Q.5 (a) Let V be an inner product space and $x, y \in V$. Prove that $x \perp y$ if and only if $\|x + y\|^2 = \|x\|^2 + \|y\|^2$. [7]

(b) Let V be an inner product space and $x, y \in V$. Prove that $x - y \perp x + y$ if and only if $\|x\| = \|y\|$. [7]

OR

Q.5 (a) Obtain orthogonal set from the set { (1, 1, 1, 1), (1, 2, 0, 1), (2, 2, 4, 0) } by using Gram – Schmidt process. [14]
