M.sc mathematics April : 2014 code No : 3115

Semester – II , Paper No : 5

Complex Analysis

(1) Each question carry equal marks

(2) All questions are compulsory

Q.1 (a) Find the roots of equation $z^2-(5+i)z+8+i=0$. [7]

(b) Separate $\left(\sqrt{i}\right)^{\sqrt{i}}$ into real and imaginary parts. [7]

OR

Q.1 (a) If $\tan\frac{x}{2}=\tanh\frac{u}{2}$ prove that (i) $\sin hu=\tan x$ (ii) $\cos hu=\sec x$ [7]

(b) If $i^{A+iB}=A+iB$ prove that $A^2+B^2=e^{-\pi(4n+1)B}$.

[7]

Q.2 (a) Show that the function $f(z)=\dfrac{x^3(1+i)-y^3(1-i)}{x^2+y^2}$, $(z\neq 0)$ and $f(0)=0$ is continuous and C-R equation s are satisfied at the origin , but is not analytic at origin. [7]

(b) Show that the function $w=\sin z$ is analytic. [7]

OR

Q.2 (a) Show that an analytic function with constant modulus is constant. [7]

(b) If $f(z)=u+iv$ and $u-v=e^x(\cos y-\sin y)$ where $f(z)$ is analytic then find $f(z)$. [7]

Q.3 (a) Obtain polar form of C-R equations. [7]

(b) Prove that every convergent sequence is a Cauchy sequence. [7]

OR

Q.3 (a) Find the radius of convergence for the following series [7]

(i) $\sum_{n=0}^{\infty} z^n$ (ii) $\sum_{n=1}^{\infty}\dfrac{(z-i)^n}{n}$

(b) State and prove Fundamental theorem of algebra. [7]

Q.4 (a) State and prove Cauchy's integral formula. [7]

(b) State and prove Morera's theorem. [7]

OR

Q.4 (a) State and prove Liouville's theorem. [7]

(b) Evaluate $\oint_C \dfrac{z+2}{z}dz$, where C is the semi circle $|z|=2$. [7]

Q.5 (a) Find the fixed points of following transformations [7]

(i) $w=\frac{z-1}{z+1}$ (ii) $w=-\frac{2z+4i}{iz+1}$

(b) Find the bilinear transformation which maps the point $z=1,i,-1$ into the points $w=i,0,-i$. [7]

OR

Q.5 (a) Find the image of infinite strip (i) $\frac{1}{4}<y<\frac{1}{2}$ (ii) $0<y<\frac{1}{2}$ under the transformation $w=\frac{1}{z}$. [7]

(b) Find the residues for the following functions at the singular points [7]

(i) $f(z)=\frac{4z+8}{2z-1}$ (ii) $f(z)=\frac{5z^2-4z+3}{(z-1)(z+2)(z+3)}$