OCT-2017

M.Sc. Physics Semester – 2 Examination
Phys-C-202 - {Quantum Mechanics}
Paper Code: 4657

Time: 2Hours 30Min

MM: 70

Note: Answer all questions. Figures to the right indicate marks allotted.
All symbols have their usual meaning.

1	a)	For simple harmonic oscillator define ladder operators a and $a^{\dagger}$. Derive energy eigenvalue spectrum, $E_n = \left(n + \frac{1}{2}\right)\hbar\omega$ using a and $a^{\dagger}$.	08
	b)	Define the followings. (i) Projection operator (ii) Unitary operator (ii) Self-adjoint operator (iv) Degenerate state	04
	c)	Write two properties of Hermitian operator.	02
		OR	
1	a)	What do you mean by A-representation? With usual notation, prove that any dynamical variable $\hat{F}$ can be represented as matrix operator, and derive $(\chi)_A = [F]_A(\psi)_A$.	08
	b)	Prove that for any two abstract operators F and G, their product is written as $[FG]_A = [F]_A[G]_A$ in A-representation.	04
	c)	Define Hilbert space and Configuration space.	02
2	a)	Derive simultaneous eigenvalue spectrum for J^2 and j_z.	08
	b)	What is the spin wave function for electron ($s = ½$), if the spin component in the direction of unit vector $\hat{\boldsymbol{n}}$ has the value $\frac{1}{2}\hbar$?	04
	c)	For Pauli's matrices prove the followings. (i) $s_+\beta = \hbar\alpha$ (ii) $\sigma_+^2 = 0$	02
		OR	
2	a)	Write note on addition of angular momenta.	07
	b)	If the components of any vector $\vec{\boldsymbol{A}}$ and $\vec{\boldsymbol{B}}$ commute with those of Pauli's matrix $\vec{\boldsymbol{\sigma}}$, then prove that $(\vec{\boldsymbol{\sigma}}\cdot\vec{\boldsymbol{A}})(\vec{\boldsymbol{\sigma}}\cdot\vec{\boldsymbol{B}}) = (\vec{\boldsymbol{A}}\cdot\vec{\boldsymbol{B}}) + i\vec{\boldsymbol{\sigma}}(\vec{\boldsymbol{A}}\times\vec{\boldsymbol{B}})$.	04
	c)	Obtain matrix representation for J_+ when $j = 1$.	03
3	a)	Using Rayleigh and Schrödinger perturbation theory for discrete part of energy eigen spectrum, derive expression for energy eigenvalue and eigen function corrected upto first order in perturbation.	07
	b)	For first excited level of H-atom, obtain equation for first-order correction to energy, $W^{(1)}$, when H-atom is placed in uniform electric field of intensity $\mathcal{E}$.	07

	Wave functions are given as follows. $\vert u_{200}\rangle = \left(\frac{1}{32\pi a_0}\right)^{\frac{1}{2}}\left(2-\frac{r}{a_0}\right)e^{-\frac{r}{2a_0}}$, $\vert u_{21-1}\rangle = \left(\frac{1}{32\pi a_0}\right)^{\frac{1}{2}}\frac{r}{a_0}e^{-\frac{r}{2a_0}}\,sin\theta\, e^{-i\varphi}$, $\vert u_{210}\rangle = \left(\frac{1}{32\pi a_0}\right)^{\frac{1}{2}}\frac{r}{a_0}e^{-\frac{r}{2a_0}}\,cos\theta$, $\vert u_{211}\rangle = -\left(\frac{1}{32\pi a_0}\right)^{\frac{1}{2}}\frac{r}{a_0}e^{-\frac{r}{2a_0}}\,sin\theta\, e^{i\varphi}$.	
3	a) For constant time dependent perturbation, derive an expression for transition amplitude $a_f^{(1)}(t)$. For this case, plot and interpret the graph of transition probability versus t^2.	08
	b) Calculate the first order correction to the ground state energy of one dimensional anharmonic oscillator of mass m and angular frequency ω subjected to a potential $V(x) = \frac{1}{2}m\omega^2x^2 + bx^4$. The ground state of an oscillator is given as $\psi_0^0 = \left(\frac{m\omega}{\pi\hbar}\right)^{\frac{1}{4}} exp\left(-\frac{m\omega x^2}{2\hbar}\right)$.	06
4	a) Optimize the trial wave function $e^{-\alpha r}$ for the ground state of H-atom taking α as variation parameter. Derive ground state energy for H-atom.	07
	b) Estimate the ground state energy of one dimensional simple harmonic oscillator using Guassian trial function.	07
	OR	
4	a) Why WKB method is also known as semi classical method. Derive asymptotic solution of one dimensional Schrödinger equation.	08
	b) Outline the variation method used for obtaining approximate value of ground state energy of a system. Prove that it gives upper bound to ground state energy.	06
5	a) What is partial wave analysis? Starting with radial wave equation, viz; $\frac{1}{r^2}\frac{d}{dr}\left(r^2\frac{dR_l(r)}{dr}\right) + \frac{2m}{\hbar^2}\left[E - V(r) - \frac{l(l+1)}{2mr^2}\right]R_l(r) = 0$, derive expression for scattering amplitude, $f(\theta) = \frac{1}{k}\sum_0^\infty(2l+1)P_l(cos\theta)e^{i\delta_l}sin\delta_l$, equation.	08
	b) Define Green's function. In terms of Green's function, derive an expression for asymptotic form for total wave function.	06
	OR	
5	a) What is Born approximation? Give difference between Born approximation and partial wave analysis. Within the Born approximation derive an expression for scattering amplitude.	08
	b) Derive an expression $sin\delta_l = -k\int_{r=0}^{\infty} U(r)\, j_l^2(kr)\, r dr$, showing relation between phase shift and potential.	06