M.Sc. (Mathematics) Semester – 3 **Code: 3472**

Paper – 11: Combinatorial Analysis – I

Time: 2 ½ Hours Nov. 2014 **Total Marks: 70**

Q.1 (a) Define r-sample and r-permutation of an n-set. Also obtain their formula. [07]

(b) Prove: $\sum_{k=1}^{n} k^2 \; {}^nC_k = n\,(n+1)\;2^{n-2}$. [07]

OR

Q.1 (a) Give combinatorial proof of $\sum_{k=0}^{n} \binom{n}{k} = 2^n$. [07]

(b) Prove that the number of ordered (r_1, r_2, ..., r_k) partitions of an n-set is $\frac{n!}{r_1! r_2! \ldots\ldots r_k!}$. [07]

Q.2 (a) In usual notations prove that $\emptyset(n) = n \prod_{p/n}\left(1 - \frac{1}{p}\right)$ where n ∈ N and p is prime. [10]

(b) Find permanent of a matrix $\begin{bmatrix} 1 & -1 & 2 & -1 \\ 3 & 1 & 2 & 3 \\ -3 & 0 & 4 & 2 \end{bmatrix}$. [04]

OR

Q.2 (a) Find number of integers between 1 to 10000 which are divisible by at least one of 2, 5 and 11. [07]

(b) Obtain the formula for derangements and hence find D_6. [07]

Q.3 (a) In usual notations prove that [10]

$$Per(A) = \sum S(A_{n-m}) - \binom{n-m+1}{1} \sum S(A_{n-m+1}) + \cdots \ldots \ldots + (-1)^{m-1} \binom{n-1}{m-1} \sum S(A_{n-1})$$

Where A is a matrix of order m x n with m ≤ n.

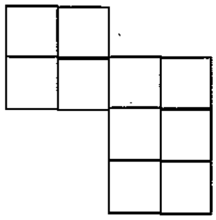
(b) Show that $n! = \sum_{r=0}^{n-1} (-1)^r \binom{n}{r} (n-r)^n$ [04]

OR

Q.3 (a) Let f(n, k) denote the number of ways of selecting k objects, no two consecutive from n objects arranged in a row then prove that [07]

$$f(n,k) = {}^{n-k+1}C_k$$

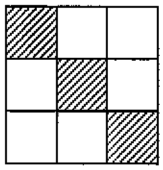
(b) Find rook polynomial of [07]



Q.4 (a) Prove that the Menage numbers U_n for n > 1 is given by [10]

$$U_n = n! - \frac{2n}{2n-1}\binom{2n-1}{1}(n-1)! + \frac{2n}{2n-2}\binom{2n-2}{2}(n-2)! + \cdots \ldots\ldots\ldots\ldots + (-1)^n \frac{2n}{n}$$

(b) Find rook polynomial of [04]



OR

Q.4 (a) Solve the following recurrence relations: [14]

(i) $a_n = a_{n-1} + a_{n-2}$, $n \geq 2$ with $a_0 = 1$ and $a_1 = 1$.

(ii) $a_n + a_{n-1} - a_{n-2} - a_{n-3} = 0$, $n \geq 3$ with $a_0 = 1$, $a_1 = 2$, $a_2 = 3$

Q.5 (a) Solve the recurrence relation $a_n = 3a_{n-1} - 2a_{n-2}$, $n \geq 2$ with initial conditions $a_0 = 1$ and $a_1 = 2$. [07]

(b) Prove in usual notations: N(q, r) = q. [07]

OR

Q.5 (a) State Ramsey's theorem. Assume that the Ramsey's theorem is true for t = 2 and show that it is true for t = 3. [10]

(b) State Pegion – Hole principle. [04]

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