

OCT - NOV - 2017.

M. Sc. (Statistics) (Semester - III) Examination
Paper - X : Linear Models & Design of Experiments

546 code - 3575

TIME: 2.5 HOURS

MARKS: 70

- Notes:- (i) Attempt All FIVE questions.
(ii) Each question carries equal marks.

1(a) State & prove Gauss - Markoff's theorem.

(b) Let $Y_i; i=1,2,3$ are independent observation with common variance σ^2 and expected values

$$E(y_1) = \theta_1 + \theta_3$$

$$E(y_2) = \theta_2 + \theta_3$$

$$E(y_3) = \theta_1 + \theta_3$$

then prove that $\theta_1 + \theta_2 + 2\theta_3$ is estimable and obtain BLUE of it and variance of BLUE.

OR

1 (a) In usual notations, prove that-

$$\text{i) } V(Q) = \sigma_C^2, \quad \text{ii) } V(P) = \sigma_D^2, \quad \text{and iii) } \text{Cov}(P, Q) = -\sigma_D^2$$

(b) Let $y_i, i = 1, 2, 3$. are three observations for which,

$$E(y_1) = \theta_1 + \theta_2, \quad E(y_2) = \theta_1 + \theta_3, \quad E(y_3) = \theta_1 + \theta_4 \text{ are given.}$$

i) Show that $\underline{b}'\theta$ is estimable if $b_1 = b_2 + b_3 + b_4$.

Also, Show that $\underline{b}'\theta$ is unique.

2 (a) Derive the intra block analysis of BIBD.

(b) Define Symmetric BIBD. Prove that for a symmetric BIBD number of common treatments between any two block is λ .

OR

2 (a) Given C-matrix of a block design explain how you will identify that whether the design is (i) connected, and (ii) balanced. Identify the below given design.

(1,2,4); (2,3,5); (3,4,6); (4,5,7); (5,6,1); (6,7,2); (7,1,3). Write its all parameters, its C-matrix and then verify that block design is balanced. Give the reasons. Also obtain Eigen value of C matrix of the block design.

(b) For BIBD (v, b, r, k, λ) , show that

$$\text{(i) } NN' = (r - \lambda)I_v + \lambda E_{vv}$$

$$\text{(ii) } C = \left(\frac{\lambda v}{k} \right) [I - E_{vv}/v]$$

3 (a) Prove that necessary and sufficient condition for a block design to be balanced is that all the non zero zero eigen roots are equal.

(b). Construct a 2^4 confounded factorial experiment by confounding ACD and ABCD .. Write its all generalized confounded interactions. Give its ANOVA table.

OR

3 (a) Define main effect and interaction effect for 2^n factorial experiment. Discuss Yates procedure method to estimate all the main effect and interaction effect for 2^3 factorial experiments. Write its ANOVA table.

(b) Check whether the block design with incidence matrix

$N = \begin{bmatrix} 2 & 1 & 1 & 1 \\ 1 & 2 & 1 & 1 \\ 1 & 1 & 2 & 1 \\ 1 & 1 & 1 & 2 \end{bmatrix}$ is connected, balanced and orthogonal.

4(a) Define the concept of confounding. Give critical comparison of total and partial confounding with an example.

(b). Explain the method of confounding 2 independent interaction in 2^m design.

OR

4(a) Construct a BIBD with parameters $v = 11, b = 11, r = 5, k = 5, \lambda = 2$.

(b) Define BIBD. Prove that (i) $b \geq v$ and (ii) $\lambda(v-1) = r(k-1)$

5 (a) Explain construction of BIBD using mutual orthogonal latin square.

(b) Construct a BIBD with parameters:-

$V = b = 7, r = k = 3, \lambda = 1$, using elements of GF (7)

OR

5. (a) Explain various type of BIBD

(b) Discuss Association scheme and hence define PBIB design of m associate classes.