November-2015

# M. Sc. (Sem. III) Examination

## Statistics: Paper - IX

*(Multivariate Analysis)*

code : 3574

Time: 2.30 Hours] [Total Marks: 70

1 (a) Let $x_{\sim_1}, x_{\sim_2}, \dots\dots, x_{\sim_n}$ be a simple random sample of size n from p-variate normal distribution $N_p(\mu_\sim, \Sigma)$. Obtain maximum likelihood estimators of $\mu_\sim, \Sigma$. 07

(b) Define singular and non-singular multivariate normal distribution. Let $\underline{x_1}, \underline{x_2}$

$$\underline{x_3} \text{ are iid } N_3(\mu, \Sigma) \quad \mu'=(1,2,4)\ \Sigma = \begin{bmatrix} 2 & .5 & 4 \\ .5 & 3 & .2 \\ 4 & .2 & .6 \end{bmatrix}$$

and if $\underline{y1} = \underline{x1} + \underline{x2}$ and $\underline{y2} = \underline{x2} + \underline{x3}$ obtain the distribution of $\underline{y_1}$ *and* $\underline{y_2}$. 07

OR

1 (a) Let $\underset{\sim}{x} \sim N_p\ (\mu_\sim, \Sigma)$ and $x_\sim, \mu_\sim$ and $\Sigma$ are partitioned as :

$$x_\sim = \begin{pmatrix} x_{\sim_1} \\ \hline x_{\sim_2} \end{pmatrix} \begin{matrix} r \\ s' \end{matrix} \qquad \mu_\sim = \begin{pmatrix} \mu_{\sim_1} \\ \hline \mu_{\sim_2} \end{pmatrix} \begin{matrix} r \\ s' \end{matrix} \qquad \Sigma = \begin{pmatrix} \Sigma_{11} & \Sigma_{12} \\ \underset{r}{\Sigma}21 & \underset{s}{\Sigma}22 \end{pmatrix} \begin{matrix} r \\ s \end{matrix}$$

with r+s=p. Show that $x_{\sim_1} - \Sigma_{12}\Sigma_{22}^{-1}x_{\sim_2}$ and $x_{\sim_2}$ are independently distributed. Hence obtain the conditional distribution of $x_{\sim_1}$ given $x_{\sim_2} = x_{\sim_2}$ 07

(b) Let $\underset{\sim}{x}r\ (r = 1,2, \dots\dots, k)$ be independently distributed as $N_p(\underset{\sim}{\mu}r, \Sigma_r)$, then show that for fixed matrices $A_r$ : mxp;

$$\sum_{r=1}^{k} A_r \underset{\sim}{x}r \sim Np\left(\sum_{r=1}^{k} A_r \underset{\sim}{\mu}r, \quad \sum_{r=1}^{k} A_r \Sigma_r \overset{A'}{r}\right).$$

Hence or otherwise obtain the distribution of sample mean vector $\bar{x}_\sim$.

07

2 (a) If X (with p components) be distributed according to $N_p(\mu, \Sigma)$. Then derive the pdf of Y=CX, C is non-singular matrix. 07

(b) Obtain characteristic function of non-singular multivariate normal distribution. 07

OR

2 (a) Define multiple correlation coefficient and partial correlation coefficients. Given $X' = (x_1, x_2, x_3)$ has tri-variate normal distribution wuith mean vector $\underline{0}$ and covariance matrix as that $\Sigma = \begin{pmatrix} 1 & \rho & \rho^2 \\ \rho & 1 & \rho \\ \rho^2 & \rho & 1 \end{pmatrix}$ find the multiple correlastion coefficient of $x_1$ on $x_2$ and $x_3$. What condition must be satisfied by $\rho$ ? 08

(b) (1)Let X∽ $N_3(\mu, \Sigma)$ and $\Sigma = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 3 & 2 \\ 1 & 2 & 2 \end{bmatrix}$ Obtain (i)partial co-relation co-efficient $r_{12.3}$(ii) the multiple correlastion coefficient of $x_1$ on $x_2$ and $x_3$.

(2) If X~$N_3(\mu, \Sigma)$, $\mu'$=(6,1,4) and $\Sigma = \begin{bmatrix} 1/2 & 1/4 & 1 \\ 1/4 & 1/2 & 3/4 \\ 1 & 3/4 & 5/2 \end{bmatrix}$ Obtain E($X_1/X_2$) and V($X_1/X_2$).( Σ =Variance-covariance Metrics, $X_2 = (X_2 \;\; , X_3)$

06

3 (a) Let $x_{\sim_1}, x_{\sim_2}, \dots\dots, x_{\sim_n}$ be n iid observations from $N_p(o_\sim, \mathrm{I_p})$ population. Obtain the distribution of the $p \times p$ symmetric matrix $V = \sum_{i=0}^{n} x_{\sim i}, x'_{\sim i}$ 07

(b) Let $V_1 \sim W_p(V_1 |n_1| \Sigma)$, and $V_2 \sim W_p(V_2 |n_2| \Sigma)$ be independently distributed. Obtain distribution of $V_1 + V_2$. 07

OR

3 (a) Let $X_i \sim N_2(0, \varepsilon)$, i=1,2,3 and $\varepsilon = \begin{bmatrix} 1 & 2 \\ 2 & 9 \end{bmatrix}$ Then obtain (i) W= $\sum X_i X_i'$ (ii) Distribution of W. (iii) Write pdf of W (4)

(b) Define canonical correlation coefficients and canonical variates. In usual notation show the canonical correlations are solution of the determinental equation (10)

$$\begin{vmatrix} -\lambda\Sigma_{11} & \Sigma_{12} \\ \Sigma_{21} & -\lambda\Sigma_{22} \end{vmatrix} = 0$$

4 (a) Define Hotelling's $T^2$ statistic and derive its null sampling distribution. 07

(b) If $\Sigma = \begin{pmatrix} 1 & \rho \\ \rho & 1 \end{pmatrix}$ where $\rho>0$; find the first principal component associated with $\Sigma$ and find the percentage of total variance explained by it. 07

OR

4 (a) In usual notation prove that

$$R^2_{1.23} = \frac{r^2_{12}+r^2_{13}-2r_{13}r_{23}r_{12}}{1-r^2_{23}}$$ 07

(b) State and prove two properties of wishart distribution. 07

5 (a) Define multiple correlation coefficient and partial correlation coefficients.Giventhat

$X' = (x1, x2, x3)$ has tri-variate normal distribution wuith mean vector $\underline{0}$ and covariance matrix as

$$\Sigma = \begin{pmatrix} 1 & \rho & \rho^2 \\ \rho & 1 & \rho \\ \rho^2 & \rho & 1 \end{pmatrix}$$

find the multiple correlastion coefficient of $x_1$ on $x_2$ and $x_3$. What conditi

07

must be satisfied by $\rho$ ?

(b) State and prove invariant property of Hotelling's $T^2$ 07

OR

5 Let $V_1 \sim W_p\ (V_1\ |n_1|\ \Sigma), n \geq p\ |\Sigma| > 0$ and let 14

$$V \begin{array}{c} \begin{pmatrix} V_{11} & V_{12} \\ V_{21} & V_{22} \end{pmatrix} \begin{array}{l} r \\ s; \end{array} \\ \begin{array}{cc} r \quad & \quad s \end{array} \end{array} \qquad \Sigma \begin{array}{c} \begin{pmatrix} \Sigma_{11} & \Sigma_{12} \\ \Sigma_{21} & \Sigma_{22} \end{pmatrix} \begin{array}{l} r \\ s; \end{array} \\ \begin{array}{cc} r \quad & \quad s \end{array} \end{array} \ r + s = p$$

Show that,

i. $V_{11.2}$ and $(V_{12}, V_{22})$ are independently distributed

ii. $V_{11.2} \sim W_r\ (\Sigma_{11.2}, n\text{-}s)$

iii. $(V_{12} / V_{22}) \sim N_{r,s}\ (\beta\ V_{22}, \Sigma_{11.2}, V_{22})$

iv. $V_{22} \sim W_s\ (\Sigma_{22}, n)$

Where, $V_{11.2} = V_{11} - V_{12}\ V_{22}^{-1}\ V_{12}^{1},\ \Sigma_{11.2} = \Sigma_{11} - \Sigma_{12}\ \Sigma_{22}^{-1}\ \Sigma_{12}^{1}$

and $\beta = \Sigma_{12}\ \Sigma_{22}^{-1}$