M. Sc. Semester- IV (MATHEMATICS)

[Code : 3474]

Oct. 2016

Paper 13: **Functional Analysis-II**

Marks:70

Q-1

(a) Define the term Dual Basis. [4]

(b) Let X be a normed space over K. Let $\{x_n\}$ be a sequence in X such that $x_n \xrightarrow{w} x$ in X and $\{k_n\}$ be a sequence in K such that $k_n \to k$ in K. Prove that $k_n x_n \xrightarrow{w} kx$. [10]

OR

Q-1

(a) If $\{x_n\}$ is a sequence in a normed space X such that $x_n \xrightarrow{w} x$ in X, then show that $\{x_n\}$ is a bounded sequence in X. [8]

(b) Give an example to show that $x_n \xrightarrow{w} x$ in X does not imply $x_n \to x$ in X. [6]

Q-2

(a) Let T be a normal operator on H. Prove: x is an eigen vector of T with eigenvalue λ if and only if x is an eigen vector of T^* with eigenvalue $\bar{\lambda}$. [10]

(b) State the Spectral Theorem. [4]

OR

Q-2

Let f be a continuous real-valued function defined on [a, b] and $\varepsilon > 0$ be given. Prove that there exist a polynomial P with the real coefficient such that $|f(x) - P(x)| < \varepsilon, \forall x \in [a, b]$. [14]

Q-3

(a) Define a Banach algebra. [4]

(b) Let A denote a complex Banach algebra with identity, let $S = A \setminus G$ denote the set of all singular elements of A, and let Z denote the set of all topological divisors of zero. Prove that Z is a subset of S. [10]

OR

Q-3

(a) Define the Banach algebra $L_1(G)$, where G is a finite group. [4]

(b) Let A denote a complex Banach algebra with identity and G denote the set of all regular elements of A. Show that G is an open set in A. [10]

Q-4

(a) Let A denote a complex Banach algebra with identity and $x \in$ A. Prove that $\sigma(x) \neq \varphi$. [8]

(b) Let A be a unital Banach algebra, $n \in N$, and $x \in$ A. Prove that $\sigma(x^n) = \sigma(x)^n$. [6]

OR

Q-4

(a) Let A denote complex Banach algebra with identity 1 and let R be a Radical in A. If $x \in$ A and $r \in$ R that such $1 - xr$ is regular, then show that $1 - rx$ is regular. [10]

(b) Define the Maximal Ideal Space of a commutative Banach algebra. [4]

Q-5

(a) Let x be a normal element in a B^*-algebra A. Then show that $\|x^2\| = \|x\|^2$. [6]

(b) Prove that the maximal ideal space M is a compact Hausdorff space. [8]

OR

Q-5

State and prove the Gelfand Neumark Theorem. [14]