CODE : 8401 S.Y.B.Sc. EXAMINATION April - 2016

M-201 Advanced Calculus

TIME : 3 HOURS INSTRUCTIONS: (1) All questions are compulsory. TOTAL MARKS:100

(2) Each question carries equal marks.

Q-1

a Find the asymptote of the curve : 7M

$x^3 - 4x^2y - xy^2 + 4y^3 - x^2 + 2xy + 3y^2 = 10.$

b In which interval the curve $y = 3x^5 - 40x^3 + 2x - 10$ is concave upward and concave downward and also find its point of inflection. 7M

c Evaluate $\lim_{(x,y)\to(0,0)} \frac{2x+y}{3y-x}$ by definition if exist. 6M

OR

Q-1

a Evaluate f_x and f_y for the function $f(x,y) = \frac{x^2+y^2}{x+y}$ and determine the value of $x f_x + y f_y$. 7M

b If $u = \log(x^3 + y^3 + z^3 - 3xyz)$, then show that 7M

$(\frac{\partial}{\partial x} + \frac{\partial}{\partial y} + \frac{\partial}{\partial z})^2 u = -9(x+y+z)^{-2}$

c If $F(x,y,u,v) \equiv x^2 + y^2 + u^2 + 2v^2 - 1 = 0$ and 6M

$G(x,y,u,v) \equiv x^2 + y^2 - u^2 - v^2 - 2 = 0$, find $\frac{\partial^2 x}{\partial u^2}$ and $\frac{\partial^2 y}{\partial u^2}$.

Q-2

a If f(x,y) is homogeneous function of x and y of degree m and if its second order partial derivatives exist then 7M

$x^2 \frac{\partial^2 f}{\partial x^2} + 2xy \frac{\partial^2 f}{\partial x \partial y} + y^2 \frac{\partial^2 f}{\partial y^2} = m(m-1)f(x,y)$

b Prove that $u = \tan^{-1}(\frac{x^2}{y})$ then prove that u is harmonic function of x , y . 7M

c Find extreme values of $f(x,y) = x^3 + y^3 - 124xy$ 6M

OR

Q-2

a g(x,y) is homogeneous function of x and y of degree m if and only if 7M

$x\frac{\partial g}{\partial x} + y\frac{\partial g}{\partial y} = mg(x,y)$.

b Prove that $f(x,y) = e^{x^2-y^2} \sin 4xy$ is harmonic function of x & y . 7M

c If $u_1 = \frac{yz}{x}, u_2 \frac{zx}{y}, u_3 = \frac{xy}{z}$, then show that $\frac{\partial(u_1,u_2,u_3)}{\partial(x,y,z)} = 4$ 6M

Q-3

a Find radius of curvature of the curve $y^2 = 2015x$ 7M

b Find general solution of the P.D.E : $(z^2 - 2yz - y^2)p + x(y+z)q = x(y-z)$ 7M

c Find radius of curvature of $2x^4 + 5y^4 + 3x^2y + xy - 3y^2 + 8x = 0$ at origin 6M

OR

Q-3	a	Obtain radius of curvature of curve r = 2 (1 - cosθ)	7M
	b	Find P.D.E. of $f(\frac{xy}{z}, \frac{yz}{x}) = 0$	7M
	c	Find P.D.E of (1) $(1 + a^3)z = 8\,(x + ay + b)^3$, (2) $z = axy + by^2$	6M
Q-4	a	Find equation of tangent plane and normal line to the surface : $4x^2 - 5y^2 + 7z^2 + 13 = 0$ at $(-1,4,-3)$	7M
	b	State & Prove : Duplication formula.	7M
	c	Prove : grad $\left(\frac{\phi}{\psi}\right) = \frac{\psi \text{grad}\phi - \phi \text{grad}\psi}{\psi^2}$	6M

OR

Q-4	a	Prove :div(f ×g) = g.curlf −f. curlg. for f = (f_1, f_2, f_3) and g = (g_1, g_2, g_3).	7M
	b	Obtain relation between beta and gamma function and hence prove that β(m ,n) = β(m+1 ,n) + β(m, n +1)	7M
	c	Evaluate : $\int_0^1 y^{q-1}\left(\log\frac{1}{y}\right)^{p-1} dy$	6M
Q-5	a	Verify Stoke's theorem for f = (y^2, xy, zx) where S is the hemisphere $x^2 + y^2 + z^2 = 1$, $z \geq 0$and C is boundary.	7M
	b	Evaluate : $\iint_s$ f · n ds where f = (yz, zx, xy) & S is the surface of the faces of the cube bounded by x = 0 , y = 0 , z = 0, x = 1 , y = 1 , z = 1.	7M
	c	Evaluate : $\int_0^1 \int_x^{\sqrt{x}} (x^2 - y^2) dxdy$	6M

OR

Q-5	a	Evaluate $\int x\, dy - y\, dx$ over circle x=cost,y=1+sint, $(-\frac{\pi}{2} \leq t \leq 0)$, From (0,0) to (1,1)	7M
	b	Evaluate : $\iiint_v$ xyz dxdydz , where v = { (x, y , z) / x , y, z ∈ [0 , 1] }	7M
	c	Evaluate : $\int_0^2 \int_1^{x^2} (x^2y - xy^2) dxdy$.	6M