CODE : 8401 S.Y.B.Sc. EXAMINATION MARCH/APRIL 2017

M-201 Advanced Calculus

TIME : 3 HOURS INSTRUCTIONS: (1) All questions are compulsory. TOTAL MARKS:100

(2) Each question carries equal marks.

Q-1 a In which interval the curve $x^2y - x^3 + y = 0$ is concave upward and concave downward and also find its point of inflection. 7M

b If $F(x, y, r, \theta) \equiv x - r\cos\theta = 0$ and $G(x, y, r, \theta) \equiv y - r\sin\theta = 0$, find $\frac{\partial r}{\partial x}, \frac{\partial r}{\partial y}, \frac{\partial \theta}{\partial x}$ and $\frac{\partial \theta}{\partial y}$. 7M

c Evaluate $\lim_{(x,y)\to(2,1)} \frac{2x+y}{2y-x}$. 6M

OR

Q-1 a If $z = f(x, y)$ and $x = e^{-u} + e^{v}$, $y = e^{u} + e^{-v}$, then prove that $z_u - z_v = y\, z_y - x\, z_x$. 7M

b Find asymptotes of curve, $x^3 + 2x^2y - xy^2 - 2y^3 + x^2 - y^2 - 3 = 0$. 7M

c Find double points of $x^3 + x^2 - 4y^2 = 0$and discuss types of double points. 6M

Q-2 a Expand $f(x, y) = \frac{y^2}{x^3}$ upto second degree in (x-1),(y+1) 7M

b Find extreme values of $f(x, y) = x^3 + y^3 - 120xy$ 7M

c If $F(x, y, u, v) \equiv x^2 + y^2 + u^2 + 2v^2 - 1 = 0$ and $G(x, y, u, v) \equiv x^2 + y^2 - u^2 - v^2 - 2 = 0$, find $\frac{\partial^2 x}{\partial u^2}$ and $\frac{\partial^2 y}{\partial u^2}$. 6M

OR

Q-2 a f(x,y) is homogeneous function of x and y of degree n if and only if $x\frac{\partial f}{\partial x} + y\frac{\partial f}{\partial y} = n\, f(x, y)$. 7M

b If $F(x, y, u, v) \equiv x^3 + y^3 + u^3 + 2v^3 - 5 = 0$ and $G(x, y, u, v) \equiv 2x^3 - y^3 + 3u^3 - v^3 - 7 = 0$, find $\frac{\partial u}{\partial x}, \frac{\partial x}{\partial u}, \frac{\partial^2 u}{\partial x^2}, \frac{\partial^2 x}{\partial u^2}$ 7M

c If $u = \sin(\sqrt{x} + \sqrt{y}) \Rightarrow x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = \frac{1}{2}(\sqrt{x} + \sqrt{y})\cos(\sqrt{x} + \sqrt{y})$ 6M

Q-3 a Find general solution of the P.D.E : $x^3 p + y^3 q = (x^2 + xy + y^2)z$. 7M

b Obtain formula of radius of curvature of curve : $y = f(x)$. 7M

c Find the radius of curvature of $2x^3 - 3x^2y + 4xy^2 - y^3 + 5x^2 - 7xy - 8y = 0$ at origin 6M

OR

Q-3 a Find solution of the P.D.E : $\frac{dx}{2017y - 2016z} = \frac{dy}{2015z - 2017x} = \frac{dz}{2016x - 2015y}$ 7M

b Find P.D.E. of $z = xy + F(x^3 + y^3)$. 7M

c Find radius of curvature $x = m\cos t + n\sin t$, $y = m\sin t - n\cos t$. 6M

Q-4	a	Prove : div(curl f) = 0 for f = (x^2yz, xy^2z, xyz^2).	7M
	b	State & Prove : Duplication formula.	7M
	c	Find equation of tangent line and normal plane of $x^2 - 2y^2 + 3z^2 = 81, 2x + y - 3z = 8$ at point (2 , 3 , 1)	6M

OR

Q-4	a	Find equation of tangent plane and normal line to the surface : $4x^2 - 5y^2 + 7z^2 + 13 = 0$ at $(-1,4,-3)$	7M
	b	Obtain relation between beta and gamma function and hence prove that $\beta(m,n) = \beta(m+1,n) + \beta(m,n+1)$	7M
	c	Prove : div(curl f) = 0 for f = (x+y ,y+z ,z+x).	6M
Q-5	a	Evaluate $\iint_s$ f · n ds where f = $(x + y^2, -2x, 2yz)$ and S is surface of plane 2x + y + 2z = 6 in the first octant.	7M
	b	Evaluate : $\iint_s xy(x^3 - y^3)dxdy$ Where $s = [0,2016] \times [0,2017]$.	7M
	c	Evaluate : $\int_0^{-\frac{\pi}{2}} \int_{-1}^{1} (x\sin y - ye^x)dy\,dx$	6M

OR

Q-5	a	$\int_c (xydx + yzdy + zx\,dz)$ where C is defined by r = (t, t^2, t^3) , $-1 \le t \le 1$.	7M
	b	Verify Green's theorem for $\int (x+y)dx + (x-y)\ dy$ where C is the ellipse $\frac{x^2}{9} + \frac{y^2}{16} = 1$	7M
	c	Evaluate: $\iiint_s xyz\,dxdydz$ Where s = [0,10]×[0,20]×[0,30].	6M