Code: 8403 S.Y. B.Sc. Examination

M-203: Numerical Analysis

Time: 3 Hours April - 2016 Total Marks: 100

Instruction: (1) All questions are compulsory.
(2) Each question carries equal marks.

Q.1 (a) Prove that the n^{th}difference of a polynomial of degree n is constant and $(n+1)^{th}$ difference will be zero, where values of independent variables are supposed to be given at equal interval. [07]

(b) In usual notations prove that $\Delta\, u^{[k]} = ku^{[k-1]}$ & $\Delta\, u^{[-k]} = -ku^{[-k-1]}$. [07]

(c) Prove that $E_r \leq \sum_{i=1}^{n} \left|\frac{\partial f}{\partial x_i}\right| \left|\frac{\Delta x_i}{y}\right|$ [06]

OR

Q.1 (a) Express $y_0 = f(x_0)$ in terms of the values of $y_0, y_1, \ldots, y_n$, where $y_i = f(x_0 + hi)$. Also express y_n in terms of y_0 and its successive differences $\Delta y_0, \Delta^2 y_0, \ldots, \Delta^n y_0$. [07]

(b) If $y_0, y_1, \ldots, y_8$ are the values of $y = f(x)$ corresponding to $x_0, x_1, \ldots, x_n$, and if there is an ε error in the value of y_4, then write error table. Also verify that the sum of errors in each column is zero. [07]

(c) Find the error that may be present in the following table: [06]

x	1.1	1.2	1.3	1.4	1.5	1.6	1.7	1.8	1.9
$f(x)$	4	6	16	40	83	154	256	400	592

Q.2 (a) Derive the Newton Gregory formula for forward interpolation. [07]

(b) Derive Gauss Backward interpolation formula. [07]

(c) Find $f'(x), f''(x)$ from the following table by Newton Gregory difference formulas. [06]

x	0	2	4	6	8	10
$y = f(x)$	5	17	69	209	485	945

OR

Q.2 (a) Derive the Newton Gregory formula for backward interpolation. [07]

(b) Use Gauss forward formula to find the value of y when $x = 3.2$ form the following table: [07]

x	1	2	3	4	5
$f(x)$	2	9	28	65	128

(c) Given the following pairs of values of x and $y = f(x)$: [06]

x	1	2	4	8	10
$y = f(x)$	0	1	5	21	27

Determine the first derivative of $f(x)$ at $x = 4$ numerically.

Q.3 (a) Derive Sterling's formula. [07]

(b) Derive Lagrange's formula. [07]

(c) Given $f(4) = 54, f(8) = 362, f(12) = 714, f(16) = 1192$, find $f(9)$ by Bessel's formula. [06]

OR

Q.3 (a) Derive Everett's formula. [07]

(b) Solve the equation $x^3 - 6x - 11 = 0$ for $x \in (3,4)$ using inverse interpolation. [07]

(c) By means of Newton's divided difference formula find the value of $f(8)$ form the following table: [06]

x	4	5	7	10	11	13
$f(x)$	48	100	294	900	1210	2028

Q.4 (a) Derive general Quadrature formula for equidistance ordinates and from that derive Trapezoidal Rule. [07]

(b) Evaluate $\int_{0.4}^{1}(2e^x - 3x - 1)dx$ by using Simpson's $1/3$ rule. [07]

(c) A river is 80 feet wide. Depth d in feet at a distance of x feet from one bank is given by the following table. [06]

x	0	10	20	30	40	50	60	70	80
d	0	4	7	9	12	15	14	8	3

Find approximately the area of the cross-section using Trapazoidal method.

OR

Q.4 (a) Derive Euler-Maclaurin summation formula. [07]

(b) Evaluate $\int_{1.5}^{4.5}(x^3 - 2x + 3)dx$ by using Simpson's $3/8$ rule. [07]

(c) Evaluate $\int_0^1 \frac{dx}{1+x^2}$ by using Weddle's rule. [06]

Q.5 (a) Discuss Newton Raphson method. [07]

(b) Given $\frac{dy}{dx} = x^2 + y, y(0) = 3$, compute $y(0.02), y(0.04)$ and $y(0.06)$ using Euler's modified method. [07]

(c) Find the approximate values of the positive roots of $2\log_{10} x = 1$ correct up to six decimal by false position method. [06]

OR

Q.5 (a) Solve the $x^3 - 2x - 5 = 0$ by the method of bisection. [07]

(b) Given $\frac{dy}{dx} = \frac{y-x}{y+x}$ with the initial condition $y = 1$for $x = 0$, find $y(0.1)$ approximatly. [07]

(c) Discuss Picard's method. [06]