

M-301: ADVANCE CALCULUS

TIME : 2:30
HOURSTOTAL
MARKS: 70

INSTRUCTIONS: (1) All questions are compulsory.
 (2) Each question carries equal marks.

- Q.1 A In which intervals , $f(x) = x^4 - 6x^3 + 12x^2 + 3x + 5$ is strictly increasing [7]
 function and strictly decreasing ?
- B Find the asymptotes of $x^3 - 4x^2y + xy^2 + 4y^3 - x^2 + 2xy + 3y^2 = 10$ [7]
- OR
- Q.1 A Prove that the curve $y^2 = x(x-1)(x-2)$ has only two points of [7]
 inflexions and also find points of inflection of the curve $y = (\log x)^3$.
- B Find asymptotes of curve, $x^2y^2 + y^4 + xy^2 - 4x^2 + y + 1 = 0$ [7]
- Q.2 A Obtain relation between beta and gamma function. [7]
- B Prove : $n+\frac{1}{2} = \frac{(2n)! \sqrt{\pi}}{4^n n!}$ [7]
- OR
- Q.2 A State & Prove : Duplication formula. [7]
- B Prove : $\int_0^\infty x^{2n-1} e^{-ax^2} dx = \frac{n}{2^n}$ [7]
- Q.3 A If $u = y^{\frac{-3}{2}} e^{\frac{-x^2}{4y}}$ then prove that $\frac{1}{x^2} \frac{\partial}{\partial x} \left(x^2 \frac{\partial u}{\partial x} \right) = \frac{\partial u}{\partial y}$. [7]
- B Prove that $u = \frac{1}{\sqrt{x^2 + y^2 + z^2}}$ then prove that u is harmonic [7]
 function of x, y, z where $(x, y, z) \neq (0, 0, 0)$.
- OR
- Q.3 A Discuss continuity of $f(x, y) = \begin{cases} \frac{x \cos(x^2+y^2)}{x^2+y^2} & , (x,y) \neq (0,0) \\ 0 & , (x,y)=(0,0) \end{cases}$ [7]
- B At $(x,y)=(0,0)$
 Evaluate : $\lim_{(x,y) \rightarrow (0,0)} \frac{x^2-y^2}{x^2+y^2}$ if exists [7]

Q.4 A If $F(x, y, u, v) \equiv x^2 + y^2 + u^2 + v^2 - 1 = 0$ and [7]
 $G(x, y, u, v) \equiv x^2 + 2y^2 - u^2 + v^2 - 1 = 0$ then prove that

$$\frac{\partial^2 x}{\partial u^2} = -\frac{9u^2}{x^3} - \frac{3}{x}, \quad \frac{\partial^2 y}{\partial u^2} = -\frac{4u^2}{y^3} + \frac{2}{y}$$

B Expand $f(x, y) = e^{px} \cos qy$ in powers of x and y . [7]

OR

Q.4 A $u = \log(x^2 + y^2 + z^2) \Rightarrow x = \frac{\partial^2 u}{\partial y \partial z}, y = \frac{\partial^2 u}{\partial z \partial x}, z = \frac{\partial^2 u}{\partial x \partial y}$ [7]

B If $\tan z = \tan^{-1}\left(\frac{x^{\frac{2}{5}} + y^{\frac{1}{5}}}{x^{\frac{3}{5}} + y^{\frac{1}{5}}}\right) \Rightarrow x \frac{\partial z}{\partial x} + y \frac{\partial z}{\partial y} = \frac{-13}{6} \sin 2z$ and
 $x^2 \frac{\partial^2 z}{\partial x^2} + 2xy \frac{\partial^2 z}{\partial x \partial y} + y^2 \frac{\partial^2 z}{\partial y^2} = \frac{13}{6} \sin 2z \left(\frac{13}{3} \cos 2z + 1\right)$

Q.5 A If $f(x, y)$ is homogeneous function of x and y of degree m and if its second order partial derivatives exist then [7]

$$x^2 \frac{\partial^2 f}{\partial x^2} + 2xy \frac{\partial^2 f}{\partial x \partial y} + y^2 \frac{\partial^2 f}{\partial y^2} = m(m-1)f(x, y)$$

B Find $\lim_{(x,y) \rightarrow (0,0)} \frac{xe^x + x - 2y}{y \cos x + \sin 2x}$, if $(x, y) \rightarrow (0, 0)$ along line $y = \frac{x}{2}$. [7]

OR

Q.5 A If $z = f(u, v)$ and [7]

$$u = e^x \cos y, \quad v = e^x \sin y \Rightarrow \frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial y^2} = (u^2 + v^2) \left(\frac{\partial^2 z}{\partial u^2} + \frac{\partial^2 z}{\partial v^2} \right)$$

B If $u = \log(x^2 + y^2 + z^2)$ then prove that $x \frac{\partial^2 u}{\partial y \partial z} = y \frac{\partial^2 u}{\partial z \partial x} = z \frac{\partial^2 u}{\partial x \partial y}$. [7]