

October-2015

CODE : 3804

SEM-III EXAMINATION,

M-302: LINEAR ALGEBRA

TIME : 2:30 HOURS

TOTAL
MARKS:70

INSTRUCTIONS: (1) All questions are compulsory.
(2) Each question carries equal marks.

- Q.1 A If W_1 & W_2 are subspaces of vector space V , then $W_1 + W_2$ and $W_1 \cap W_2$ are both subspaces of V . [7]
- B If $V = \{(x, y) | x > 0, y \in \mathbb{R}\}$ and $\alpha \in \mathbb{R}$, for $a, b, c, d \in \mathbb{R}$ the function define as $(a, b) + (c, d) = (ac, b+d)$ and the scalar multiplication defined as $\alpha(a, b) = (a^\alpha, \alpha b)$ then prove that V is vector space. [7]
- OR
- Q.1 A Determine which of the following vectors are in $[S]$, where $S = \{(1, 2, 1), (1, 1, -1), (4, 5, -2)\}$. (a) $(0, 0, 0)$ (b) $(1, -3, 5)$ [7]
- B Prove that $W_1 = \{(x, y, z) | \sqrt{2}x = \sqrt{3}y\}$ is vector subspace of \mathbb{R}^3 But $W_2 = \{(x, y, z) | x^2 + y^2 + z^2 \leq 1\}$ is not vector subspace of \mathbb{R}^3 . [7]
- Q.2 A Check following sub sets of \mathbb{R}^3 is linearly dependent or not? $S_1 = \{(1, 0, 0), (1, 1, 1), (1, 2, 3)\}$ and $S_2 = \{(1, 5, 2), (0, 0, 1), (1, 1, 0)\}$. [7]
- B W_1 and W_2 are subspaces of vector space V and $W = W_1 + W_2$ for every $w \in W$, \exists unique vector such that $w = w_1 + w_2$ iff $W = W_1 \oplus W_2$ [7]
- OR
- Q.2 A If w_1 and w_2 are sub spaces of finite dimensional vector space V then $\text{Dim}(w_1 + w_2) = \text{Dim } w_1 + \text{Dim } w_2 - \text{Dim}(w_1 \cap w_2)$. [7]
- B Prove that the subsets $\{(1, \frac{2}{5}, -1), (0, 1, 2), (\frac{3}{4}, -1, 1)\}$ of \mathbb{R}^3 are its basis. [7]
- Q.3 A If $\text{Dim } V = n$ then prove that V is isomorphic to vector space \mathbb{R}^n . [7]
- B If a linear transformation T on V satisfies the condition $T^2 + I = T$ then prove that T^{-1} exists. [7]
- OR
- Q.3 A $T: \mathbb{R}^3 \rightarrow \mathbb{R}^2$; $T(x, y, z) = (x+y, y+z)$; $B_1 = \{(1, 1, 1), (1, 1, 0), (1, 0, 0)\}$ $B_2 = \{(1, 0), (0, 1)\}$ then find the matrix $[T: B_1, B_2]$. [7]
- B If $T: U \rightarrow V$, is linear map then [7]
- (1) $R(T)$ is subspace of V .
- (2) $N(T)$ is subspace of V .

- Q.4 A State and prove , Rank - Nullity theorem. [7]
 B For linear transformation $T: \mathbb{R}^2 \rightarrow \mathbb{R}^3$ defined as $T(x,y) = (x, x+y, y)$; for every $(x,y) \in \mathbb{R}^2$, find R_T, N_T & $n(T), r(T)$ [7]
- OR
- Q.4 A Prove that parallelogram is rhombs iff the diagonals are perpendicular to each other. [7]
 B $T: \mathbb{R}^4 \rightarrow \mathbb{R}^3$ is linear transformation [7]
 $T(x_1, x_2, x_3, x_4) = (x_1 + x_2, x_2 - x_4, x_3 - x_4)$ then find $N_T, R_T, n(T)$ and $r(T)$
- Q.5 A In inner product space, Prove that triangle is right angle if \exists one side whose square equal to the sum of square of other sides. [7]
 B state and prove Schwarz's inequality [7]
- OR
- Q.5 A If x and y are vectors in unitary space then show that, [7]
 $4 \langle x, y \rangle = \|x + y\|^2 - \|x - y\|^2 + i \|x + iy\|^2 - i \|x - iy\|^2$
 B State and prove triangle inequality. [7]