- 3 APR 2019

B.Sc. SEM - III EXAMINATION:

PAPER NO.:MAT-CC-303

CODE NO: 20677/20895

TIME:2:30 HOURS

INSTRUCTIONS (1) ALL QUESTIONS ARE COMPULSORY.

(2) EACH QUESTION CARRY EQUAL MARKS

TOTAL MARKS:70

Q.1 A Prove: If $f'(x) < 0,\ \forall x \in R$ then f(x) is strictly decreasing function and hence $f(x) = \frac{\sin x}{x}$ is strictly decreasing function in $\left(0, \frac{\pi}{2}\right)$. [7]

B Find asymptotes of : $x^3 - xy^2 + y^2 = 0$ [7]

OR

Q.1 A In which intervals, the curve, $y = x^3 - 3x^2 + 5x + 1$ is concave upwards and concave downwards. Also find point of inflexion. [7]

B Discuss method of the asymptotes of general algebraic curve. [7]

Q.2 A Evaluate : $\lim_{(x,y)\to(0,0)} \frac{x^{20}-y^{20}}{x^{30}+y^{30}}$ if exists. [7]

B Discuss continuity of $f(x,y) = \frac{x^2-y^2}{x+y}$, (x,y)≠(0,0)

$= 0$, (x,y)=(0,0) at (x,y)=(0,0) [7]

OR

Q.2 A By definition Evaluate : $\lim_{(x,y)\to(2,1)} \frac{2x+y}{3y-x}$ if exists. [7]

B Evaluate f_x and f_y by definition for $f(x,y) = \frac{x(x-y)}{x+y}$, x+y ≠ 0 [7]

$= 0$, x+y = 0

and hence obtain $xf_x + yf_y$

Q.3 A If $u = \log(x^2 + y^2 + z^2)$, then prove that $x\frac{\partial^2 u}{\partial y\,\partial z} = y\frac{\partial^2 u}{\partial z\,\partial x} = z\frac{\partial^2 u}{\partial x\,\partial y}$ [7]

B If $u = x + y + z, v = x^2 + y^2 + z^2, w = x^3 + y^3 + z^3 - 3xyz$ [7]

$\Rightarrow \frac{\partial(u,v,w)}{\partial(x,y,z)} = 0$

OR

Q.3 A If f is differentiable homogeneous function of two variable x and y of degree n $\Leftrightarrow xf_x + yf_y = n\,f(x,y)$ [7]

B If $u = \log(x^2 + y^2 + z^2)$ then prove that $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} = \frac{2}{x^2+y^2+z^2}$ [7]

Q.4 A Expand f(x,y) = logxy in the powers of x -1and y -1 [7]

B Find extreme values of $2(x - y)^2 - x^4 - y^4$ [7]

OR

Q.4 A Expand f(x,y) = $e^{ax}\sin by$ in the powers of x and y. [7]

B Find extreme values of $x^3 + y^3 - 3x - 12y + 20$ [7]

Q.5	A	State & Prove : Duplication formula.	[7]
	B	Obtain equation of tangent plane and normal line of surface $x^2+ y^2+z^2 = 49$ at point (6,2,3)	[7]

OR

Q.5	A	Evaluate : $\int_0^1 y^{q-1}\left(\log\frac{1}{y}\right)^{p-1} dy$	[7]
	B	State & Prove: Relation between Beta & Gamma Function .	[7]