

21 OCT 2019

Examination October -2019

Seat No.

B.SC.SEM- III

Mathematics: Paper no. MAT-CC -303 CODE: 20677/20895

ADVANCED CALCULUS -I

Total marks - 70

Time : 2:30 Hours

Instruction: All questions are compulsory.

Q-1 A Prove: If function f is defined on [a,b] and $f'(x) > 0, \forall x \in (a, b)$ then f is strictly increasing on (a,b) and hence prove that $f(x) = \frac{\tan x}{x}$ is strictly increasing on $(0, \frac{\pi}{2})$ 14

OR

Q-1 A(i) Find points of inflexion of $y(1+x^2) = x^3$ 07

A(ii) Find the asymptotes of the curve $x^3 + 3x^2y - 4y^3 - x + y + 2 = 0$ 07

Q-1 B Attempt any three. C Pluson ni 2000 2000 03

(i) Define: An increasing function.

(ii) Point of inflexion of $y = x^3$ is(Fill the blank) शुद्धतः ० शुद्धतः

(iii) Prove: Sinx is an increasing function in first quadrant.

(iv) True or false: Function $y = e^x$ has point of inflexion.

(v) True or false: If $f'(x) = 0$ for all x in domain of f then f is constant function. 14

Q-2 A If $u = \log(x^3 + y^3 + z^3 - 3xyz)$ then prove that

$(\frac{\partial}{\partial x} + \frac{\partial}{\partial y} + \frac{\partial}{\partial z})^2 u = -9(x^3 + y^3 + z^3)^{-2}$ and if $u = \tan^{-1} \frac{x+y}{x-y}$ then prove that

$xu_x + yu_y = 0$

OR

Q-2 A(i) If $u = x + y + z, v = x^2 + y^2 + z^2, w = x^3 + y^3 + z^3 - 3xyz \Rightarrow \frac{\partial(u,v,w)}{\partial(x,y,z)} = 0$ 07

A(ii) If $u = y^{\frac{-3}{2}} e^{\frac{-x^2}{4y}} \Rightarrow \frac{1}{x^2} \frac{\partial}{\partial x} (x^2 \frac{\partial u}{\partial x}) = \frac{\partial u}{\partial y}$ 07

Q-2 B Attempt any three. 03

(i) If $f(x,y) = \log(x^2 + y^2)$ then $f_{xy} = \dots\dots$ (Fill the blank)

(ii) If $u = x + y, v = x^2 + y^2$ then $\frac{\partial(u,v)}{\partial(x,y)} = \dots\dots$ (Fill the blank)

(iii) If $u = \log(x^2 + y^2)$ then $u_{xx} + u_{yy} = \dots\dots$

(A) 0 (B) 1 (C) 2x+2y

(D) none of these.

(iv) If $u = x^2 + y^2$ then u_{xy} at (1,1) =

(A) 0 (B) 1 (C) -2

(D) none of these.

(v) True or false: If $y = f(x)$ then $\frac{dy}{dx} = \frac{\partial y}{\partial x}$