B. Sc Semester-III Mathematics Examination March/April-2016
Advanced Calculus
Paper No: M-301

Total Marks:70

Time : 2:30 hours
Paper code: 3803

Instructions:

(1) All questions are compulsory.
(2) Each question carry equal marks.

Que-1. (a) Find asymptotes parallel to coordinate axis of the curve $x^2y - 3x^2 - 5xy + 6y + 2 = 0$. [7]
(b) Let $f : \mathbb{R} \to \mathbb{R}$ be a differentiable function. If $f'(x) > 0$ for all $x \in \mathbb{R}$, then prove that f is an increasing function. [7]

or

Que-1. (a) Show that the parabola $y^2 = 4ax$ has no asymptotes. [7]
(b) Examine the concavity of the function $f(x) = 5x^3 + 2x^2 - 3x$ and find its point(s) of inflexion. [7]

Que-2. (a) State and prove duplication formula. [7]
(b) Prove that $\int_0^{\infty} \sqrt{x}e^{-3\sqrt{x}}dx = \frac{315}{16}\sqrt{\pi}$. [7]

or

Que-2. (a) State and prove the relation between beta and gamma function. [7]
(b) Define *beta function*. In usual notation prove that
(i) $\beta(p,q) = \beta(q,p)$,
(ii) $\beta(p,q) = \int_0^{\infty} \frac{y^{p-1}}{(1+y)^{p+q}}dy$. [7]

Que-3. (a) Examine the continuity of the function
$$f(x,y) = \begin{cases} \frac{xy}{x^2+y^2}, & (x,y) \neq (0,0) \\ 0, & (x,y) = (0,0) \end{cases}$$ at the point $(0,0)$. [7]
(b) If $u = \sin^{-1}\left(\frac{x}{y}\right) + \tan^{-1}\left(\frac{y}{x}\right)$, then find the value of $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y}$. [7]

or

Que-3. (a) If $f(x,y) = \frac{x+y}{2x-y}$, then show that $\lim\limits_{x\to 0}[\lim\limits_{y\to 0} f(x,y)] \neq \lim\limits_{y\to 0}[\lim\limits_{x\to 0} f(x,y)]$. [7]
(b) If $u = \log(x^3 + y^3 + z^3 - 3xyz)$, then show that $\left(\frac{\partial}{\partial x} + \frac{\partial}{\partial y} + \frac{\partial}{\partial z}\right)^2 u = -\frac{9}{(x+y+z)^2}$. [7]

Que-4. (a) If $u(x,y,z) = \frac{1}{x^2+y^2+z^2}$, then find the value of $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2}$. [7]
(b) Expand $e^x \sin y$ in powers of x and y near the point $(0,0)$. [7]

or

Que-4. (a) If $x + y + z = u$, $y + z = uv$ and $z = uvw$, then show that $\frac{\partial(x,y,z)}{\partial(u,v,w)} = u^2v$. [7]
(b) Obtain Taylor's expansion of $\tan^{-1}\left(\frac{y}{x}\right)$ about $(1,1)$ upto second degree terms.

Que-5. (a) State and prove Taylor's theorem. [7]

(b) If u and v are functions of x and y, then prove that $\frac{\partial(u,v)}{\partial(x,y)} \cdot \frac{\partial(x,y)}{\partial(u,v)} = 1$. [7]

or

Que-5. (a) State and prove Euler's theorem. [7]
(b) Find the extreme values of function $x^3 + y^3 - 3axy$. [7]