

Examination October -2019

Seat No.

B.SC.SEM- III

Mathematics: Paper no. MAT-CC -304 CODE: 20678/20896

LINEAR ALGEBRA –I AND NUMERICAL METHODS-I

Time : 2:30 Hours

Total marks - 70

Instruction: All questions are compulsory.

- Q-1 A If W_1 and W_2 be two subspaces of a finite dimensional vector space V then ,
 $\text{Dim}(W_1 + W_2) = \text{Dim}(W_1) + \text{Dim}(W_2) - \text{Dim}(W_1 \cap W_2)$ 14
 OR
- Q-1 A(i) If W_1 & W_2 are sub spaces of vector space V ,then $W_1 + W_2$ is also a sub space of V . 07
 A(ii) Set of vectors $\{v_1, v_2, \dots, v_n\}$ is L.D iff there exist a vector v_k , ($2 \leq k \leq n$), which is
 Linear combination of its preceding vectors v_1, v_2, \dots, v_{k-1} 07
- Q-1 B Attempt any three. 03
- (i) If W_1 & W_2 are sub spaces of vector space V then is a sub space of V .
 (A) $W_1 + W_2$ (B) $W_1 \cap W_2$ (C) $W_1 - W_2$ (D) All of these.
- (ii) True or false: Any non empty subset of L.I. set is always L.I. set.
- (iii) True or false: Any super set of L.D. set is always L.D. set.
- (iv) Define: Basis of vector Space.
- (v) If W is subspace of vector space V then for $\forall \alpha, \beta \in R$, $\overline{w_1}, \overline{w_2} \in W$
 (A) $\overline{w_1} + \overline{w_2} \in W$ (B) $\alpha \overline{w_1} + \beta \overline{w_2} \in W$ (C) $\alpha \overline{w} \in W$ (D) All of these.
- Q-2 A State and prove Rank – Nullity Theorem. 14
 OR
- Q-2 A(i) For Linear transformation $T: U \rightarrow V$, if $U = \text{SP}\{u_1, u_2, \dots, u_n\}$ then prove that
 $R_T = \text{SP}\{T(u_1), T(u_2), \dots, T(u_n)\}$ (where SP is defined as Span of a vector set) 07
- A(ii) Prove: For linear transformation $T: U \rightarrow V$ is one-one iff $N_T = \{\theta\}$ 07
- Q-2 B Attempt any three. 03
- (i) If $T: U \rightarrow V$ is linear transformation then
 (A) $R_T \subset U$ (B) $R_T \subset V$ (C) $N_T \subset R_T$ (D) All of these.
- (ii) If $T: U \rightarrow V$ is linear transformation and $N_T = \{\theta\}$ then Which one of following is not true?
 (A) $\text{Dim } U = \text{Dim } V$ (B) $\text{Dim } U \neq \text{Dim } V$ (C) $\text{Dim } N_T = 0$ (D) $R_T = V$
- (iii) Define: Linear transformation
- (iv) True or false: If $\text{Dim } U = m$ and $\text{Dim } V = n$ then $\text{Dim } L(U, V) = mn$.
- (v) If $T: R^2 \rightarrow R^2$ is one-one and on to linear transformation then $\text{Dim } R_T = \dots$
 (Fill the blank)
- Q-3 A State and prove Polynomial in factorial notation. 14
 OR
- Q-3 A(i) If $y = 7x^4 - 5x + 2$ then find E_a, E_r, E_p in y at $x=2$ if error in x is 0.12 07
 A(ii) State and prove formula for Estimation of Errors. 07

- Q-3 B Attempt any four. 04
- (i) In usual notation prove that $\mu^2 = 1 + \frac{1}{4}\delta^2$
 - (ii) Define: Absolute error
 - (iii) Find value of $\delta^5 y_8$
 - (iv) $E = \dots$
 (A) $1 + \Delta$ (B) $\Delta \nabla$ (C) $\Delta + \nabla$ (D) $\nabla - \Delta$
 - (v) $\Delta x^{[n]} = \dots$ (fill the blank)
 - (vi) $E f(x) = \dots$
 (A) $f(x+h)$ (B) $f(x-h)$ (C) $f(x+nh)$ (D) none of these
- Q-4 Derive: Newton- Gregory forward difference formula. 14
- OR
- Q-4 A(i) Derive Gauss's forward difference formula. 07
- A(ii) Derive derivatives of Newton- Gregory backward formula. 07
- Q-4 B Attempt any four. 04
- (i) Write formula of Gauss's backward difference formula.
 - (ii) Make difference table for

x	0	1	2	3	4
y	2	3	18	83	258
 - (iii) Write formula of Newton- Gregory backward difference formula.
 - (iv) In forward difference formula value of $P = \dots$ (fill the blank)
 - (v) Relation between E and D is
 - (a) $E = e^{-hD}$ (b) $E = e^{hD}$ (c) $E = \log h D$ (d) $E = \log D$
 - (vi) $hD = \dots$ (fill the blank)