T.Y.B.Sc. EXAMINATION:MARCH/APRIL 2017

PAPER NO.:M-301
TIME:3 HOURS

ABSTRACT ALGEBRA

INSTRUCTIONS(1)ALL QUESTIONS ARE COMPULSORY.
(2)EACH QUESTION CARRY EQUAL MARKS

CODE NO: 8961
TOTAL MARKS:100

Q.1	A	State and prove that Lagrange's theorem for finite group.	[7]
	B	$G = \{1, 2, 3, 4, 5, 6\}$ prove that $(G, \cdot_7)$ is a commutative group.	[7]
	C	For $\sigma \in S_7$, $\sigma = (1\ 3\ 5\ 7)$ find σ^{-1} and $O(\sigma)$.	[6]
		OR	
Q.1	A	State and prove cancellation laws in group $(G, *)$	[7]
	B	Prove that $(Z_7, +_7)$ is commutative group.	[7]
	C	In a group G for a , b $\in$ G if $a^5 = e$, $aba^{-1} = b^2$ then prove that $O(a) = 31$	[6]
Q.2	A	N is normal subgroup of group G then prove that $N_a N_b = N_{ab}$	[7]
	B	G is cyclic group of order 24 generated by a and $H = \langle a^6 \rangle$ obtain factor group G/H of H in G.	[7]
	C	Prove that fourth root of unity is acyclic group with respect to multiplication.	[6]
		OR	
Q.2	A	State and prove that Fundamental theorem of homomorphism.	[7]
	B	If $f : G \to G'$ is an isomorphism and if H is normal subgroup of G then f(H) is normal subgroup of G'.	[7]
	C	Obtain all cyclic subgroup of $(Z_{12}, +_{12})$ and its lattice diagram.	[6]
Q.3	A	Prove that $(Z_7, +_7, \cdot_7)$ is a field.	[7]
	B	$(R, +, \cdot)$ is a ring and S is non empty subset of R. S is sub-ring of ring R if and only if $a - b \in S$ and $a \cdot b \in S, \forall a, b \in S$.	[7]
	C	Show that the characteristics of an integral domain D must be either zero or prime p .	[6]
		OR	
Q.3	A	Define: Boolean ring and prove that it is commutative ring.	[7]
	B	$(R, +, \cdot)$ is a ring and $S = \{ m + n\sqrt[3]{2} + p\sqrt[3]{4} \ / \ m, n \in Z \}$ prove that S is a sub-ring of ring R.	[7]
	C	n is a positive integer and m is a non-zero element of $(Z_n, +_n, \cdot_n)$	[6]
Q.4	A	Prove: a commutative ring R with unity is a field if it has no proper ideal.	[7]
	B	Let R be a commutative ring and let $a \in R$.prove that the $I = \{ x \in R \ / \ ax = 0 \}$ is an ideal.	[7]
	C	Prove : an ideal $\langle p \rangle$ in PID is maximal if and only if p is an irreducible.	[6]
		OR	
Q.4	A	Prove that field has no proper ideal.	[7]
	B	State and prove Fermat's theorem.	[6]
	C	Find all principal ideals and maximal ideals of ring $(Z_{12}, +_{12}, \cdot_{12})$	[7]
Q.5	A	Prove that the degree of product of two non-zero polynomials is equal to sum of their degree.	[7]
	B	State and prove factor theorem for polynomial f(x) in F[x].	[7]
	C	Find $(i + j)(1 + i + j - k)^{-1}$.	[6]
		OR	
Q.5	A	State and prove division algorithm for polynomials in F[x].	[7]
	B	Find sum and product for polynomials f = (2 , 1 ,-2 ,-3 ,0 ,4 , 0, 0,......) and g = (1, -4, 2 , -3 ,0 , 0,......).	[7]
	C	Prove that $f(x) = x^3 + 3x + 2$ is irreducible over $z_5[x]$.	[6]