T.Y.B.Sc. EXAMINATION: Oct - 2017

PAPER NO.:M-301
TIME:3 HOURS

ABSTRACT ALGEBRA

CODE NO: 8961
TOTAL MARKS:100

INSTRUCTIONS(1)ALL QUESTIONS ARE COMPULSORY.
(2)EACH QUESTION CARRY EQUAL MARKS

Q.1 A In a group G , G is a commutative group if and only if $(ab)^{-1} = a^{-1}b^{-1}\ \forall\ a,b \in G$. [7]

B G is a group and $a \in G$ is fixed element .H = { $x \in G$/ xa = ax }.prove that $H \leq G$. [7]

C For $\sigma \in S_8$,σ = (1 3 7 8)find σ^{-1} and O(σ). [6]

OR

Q.1 A State and prove that necessary and sufficient condition for non empty subset of group is to be a subgroup. [7]

B State and prove that Langrange's theorem for finite group. [7]

C In a group G for a , $b \in G$ if $O(b) = 4$, $aba^{-1} = b^2$ and if O(a) is an odd number then find O(a). [6]

Q.2 A G is a group and $g \in G$ is fixed element. prove that $i_g : G \to G$, $i_g(x) = gxg^{-1}$ is an isomorphism. [7]

B (G , +) = (Z , +) and (H , +) = (5Z , +) obtain factor group G/H of H in G. [7]

C Prove that intersection of two normal subgroups of group G is normal subgroup. [6]

OR

Q.2 A State and prove cayle's theorem. [7]

B G is cyclic group of order 18 generated by a and H = < a^3> obtain factor group G/H of H in G. [7]

C G = S_3 and H = { ρ_0 , ρ_1 , ρ_2 } prove that H is a normal subgroup of G. [6]

Q.3 A Prove that (Z_5 , $+_5$,$\cdot_5$) is a field. [7]

B Define: Boolean ring and prove that it is commutative ring. [7]

C Prove that every field is an integral domain. [6]

OR

Q.3 A Prove that finite integral domain is a field. [7]

B Prove that (Z_7 , $+_7$,$\cdot_7$) is a commutative ring with unity. [7]

C The cancellation laws hold in ring R if and only if R has no zero divisors. [6]

Q.4 A Prove every PID is a UFD. [7]

B Using Fermat's theorem, prove that $n^{37} - n$ is divisible by 19 and also find reminder when 13^{47} is divided by 19. [7]

C Prove : an ideal < p >in PID is maximal if and only if p is an irreducible. [6]

OR

Q.4 A R is commutative ring with unity. An ideal M of R is maximal if and only if R/M is a field. [7]

B Using Euler's theorem , obtain reminder when 7^{1000} is divided by 24. [7]

C $\emptyset : R \to R'$ is isomorphism if R is an ideal of R then $\emptyset(I)$ is an ideal of R'. [6]

Q.5 A State and prove division algorithm for polynomials in F[x]. [7]

B For an integral domain D, prove that D[x] is also an integral domain with respect to the binary operations of addition and multiplication of polynomial. [7]

C Any ideal in integral domain F[x] is a principal ideal. [6]

OR

Q.5 A Prove that the degree of product of two non-zero polynomials is equal to sum of their degree. [7]

B State and prove reminder theorem for polynomials in F[x]. [7]

C Find all irreducible polynomials in $z_2[x]$ and in $z_3[x]$ of degree 2 or 3. [6]