CODE :8965

T.Y.B.Sc. EXAMINATION April - 2016

M-305 COMPLEX ANALYSIS

TIME : 3 HOURS   INSTRUCTIONS: (1) All questions are compulsory.   TOTAL

(2) Each question carries equal marks.   MARKS:100

Q-1 a State & prove De' moivre's theorem 7M

b Expand : $\cos^n\theta$ in terms of $\cos\theta$ 7M

c Expand : $\tan\alpha$ in terms of α 6M

OR

Q-1 a If $x = \text{cis}\,\alpha$, $y = \text{cis}\,\beta$, then prove that 7M

(1) $\frac{x-y}{x+y} = i\tan\frac{(\alpha-\beta)}{2}$ (2) $\frac{(x+y)\ (xy-1)}{(x-y)(xy+1)} = \frac{\sin\alpha+\sin\beta}{\sin\alpha-\sin\beta}$

b Expand Cos $n\theta$ and sin $n\theta$ in terms of $\cos\theta$ and $\sin\theta$. 7M

c Expand : $\sin 12\theta$ in terms of $\sin\theta$ 6M

Q-2 a find Critical points of the bilinear mappings : 7M

(1) $\omega = \frac{az+bz}{cz+d}$,ad-bc≠0 (2) $\omega = \frac{3z}{1-2z}$

b If $\tan(\log(x+iy)) = a + ib$, then prove that $\tan(\log(x^2+y^2)) = \frac{2a}{1-a^2-b^2}$. 7M

c show that composition of two mobius mappings is again mobius mapping 6M

OR

Q-2 a If $\cos^{-1}(u + iv) = x + iy$ then prove that: 7M

(1) $(u\cdot\sec x)^2-(v\cdot\text{cosec}\,x)^2= 1$ (2) $(u\cdot\sec hy)^2+(v\cdot\text{cosec}\,hy)^2= 1$

b show that the set of all bilinear mapping is a group under composition 7M

c Find zeroes of sin hz and cos hz. 6M

Q-3 a If complex function of complex variables is differentiable at a given point then it is continuous there ,but converse is not true. 7M

b If $w = (x^2 + axy + by^2) + i(cx^2 + dxy + y^2)$ is analytic function,then find a, b, c and d. 7M

c Show why function $f(z) = e^y e^{ix}$ is nowhere analytic. 6M

OR

Q-3 a Obtain C-R condition in polar form for analytic function. 7M

b If F(Z) satisfied C-R condition in polar form then prove that , 7M
$F'(Z) = (\cos\theta - i\sin\theta)(u_r + iv_r)$.

c If f(z) is analytic function of z , then prove that $\left[\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right] |\mathrm{Re} f(z)|^2 = 2 |f'(z)|^2$. 6M

Q-4 a If f(z) satisfied C-R condition in polar form then obtain Laplace equation in polar form. 7M

b Prove that $u = \frac{1}{2}\log(x^2 + y^2)$ is harmonic function and obtain its harmonic conjugate. 7M

c Prove that $u = x^2 - y^2$ & $v = \frac{-y}{x^2+y^2}$ *are both harmonic function and* $F(Z) = u + iv$ is not analytic functions. 6M

OR

Q-4 a If u and v both are harmonic functions of x and y and $C = u_y + v_x$ and $D = u_x - v_y$ then prove that f(z)= C+ iD is an analytic function. 7M

b Prove that $u = x^2 - y^2$ & $v = \frac{y}{x^2+y^2}$ are both harmonic functions even though $f(z) = u+iv$ is not analytic function. 7M

c Prove that $u = \frac{y}{x^2+y^2}$ is harmonic function . Also obtain harmonic conjugate of u and corresponding analytic function. 6M

Q-5 a State and prove Cauchy-Residue theorem. 7M

b Show that mapping $w = \frac{1}{z}$ transforms circles and lines into circles and lines. 7M

c Using Cauchy - Residue theorem prove that $\int_0^\infty \frac{x \sin ax}{x^2+l} dx = \frac{\pi}{2} e^{-al}$. 6M

OR

Q-5 a Discuss mapping $w = z^n$. 7M

b Discuss mapping $w = e^x(\cos x + i\sin x)$ and prove it is conformal. 7M

c Find value of $\int_C \frac{z+1}{(z-1)(z-2)(z^2+1)} dz$ where c: $|z| = 3$. 6M