CODE: 8965 T.Y.B.SC. MATHEMATICS EXAMINATION - Oct - 2017

M-305 COMPLEX ANALYSIS

TIME: **3 HOURS**] Instructions: (1) All questions are compulsory. [TOTAL MARKS: **100**
(2) Each question carries equal marks.

Q.1	A	Expand: $\cos n\theta$ in terms of $\cos\theta$.	7
	B	Prove $32\cos^6\theta = \cos6\theta + 6\cos4\theta + 15\cos2\theta + 10$	7
	C	Expand: $\sin x$ in terms of x.	6
		OR	
Q.1	A	State and prove De'Moivre's theorem.	7
	B	If $\sin\alpha + \sin\beta + \sin\gamma = \cos\alpha + \cos\beta + \cos\gamma = 0$, prove that $\sin 3\alpha + \sin3\beta + \sin3\gamma = 3\sin(\alpha+\beta+\gamma)$ and $\cos3\alpha + \cos3\beta + \cos3\gamma = 3\cos(\alpha+\beta+\gamma)$.	7
	C	Use De'Moivre's theorem to solve $x^4 - x^3 + x^2 - x + 1 = 0$.	6
Q.2	A	If $y = \log\tan x$, show that $\sinh ny = \frac{1}{2}(\tan^n x - \cot^n x)$.	7
	B	Prove that $\tan^{-1} x = \sinh^{-1}\frac{x}{\sqrt{1-x^2}}$.	7
	C	Separate the real and imaginary parts of $\tanh(x+iy)$.	6
		OR	
Q.2	A	If $u = \log\tan\left(\frac{\pi}{4}+\frac{\theta}{2}\right)$, prove that $\theta = -i\log\tan\left(\frac{\pi}{4}+\frac{i\pi}{2}\right)$.	7
	B	Show that $\tan^{-1} z = \frac{i}{2}\log\frac{i+z}{i-z}$.	7
	C	Separate $\tan^{-1}(x+iy)$ into real and imaginary parts.	6
Q.3	A	Obtain Cauchy-Riemann condition in Cartesian form for analytic function.	7
	B	Prove $f(z) = e^z$ is analytic function.	7
	C	Let f be an analytic function in a domain D. If $\lvert f(z)\rvert$ is constant function for all z in D, then prove that f is constant.	6
		OR	
Q.3	A	Show that the polar form of Cauchy-Riemann equations are $\frac{\partial u}{\partial r} = \frac{1}{r}\frac{\partial v}{\partial\theta}, \frac{\partial v}{\partial r} = -\frac{1}{r}\frac{\partial u}{\partial\theta}$.	7
	B	Prove that the function $f(z)$ defined by $f(z) = \frac{x^3(1+i) - y^3(1-i)}{x^2+y^2}$, $(z \neq 0), f(0) = 0$ is continuous.	7
	C	If $f(z) = e^{2z}$, prove that f(z) is entire function, also obtain f'(z) and f''(z).	6
Q.4	A	Show that real and imaginary parts of an analytic function satisfy the Laplace equation in two variables.	7
	B	Show that is $u(x,y)$ harmonic in some domain and find harmonic conjugate $v(x,y)$ when $u(x,y) = 2x(1-y)$.	7
	C	Prove that $u(r,\theta) = \text{Log}\,r$; $r > 0, 0 < \theta < 2\pi$ is harmonic function also obtain harmonic conjugate of u.	6
		OR	
Q.4	A	A function $f(z) = u(x,y) + iv(x,y)$ is analytic in domain D if and only if v is a harmonic conjugate of u.	7
	B	Show that is $u(x,y)$ harmonic in some domain and find harmonic conjugate $v(x,y)$ when $u(x,y) = 2x - x^3 + 3xy^2$.	7
	C	If u and v are both harmonic functions of x and y and $P = u_y + v_x$ and $Q = u_x - v_y$ then prove that $f(z) = P + iQ$ is an analytic function.	6

Q.5	A	Prove that $w = z^2$ is conformal $w = \bar{z}$ is not conformal.	7
	B	Find the bilinear transformation which maps the points $z = 1, i, -1$ on to the points $w = i, 0, -i$.	7
	C	Determine the poles of the function $f(z) = \frac{z^2}{(z-1)^2(z+2)}$ and the residue at each pole.	6

OR

Q.5	A	State and prove Cauchy's Residue theorem.	7
	B	Determine the bilinear transformation that maps the points $1, i, -1$ respectively in to 2, i, -2.	7
	C	Evaluate $\int_C \frac{z-3}{z^2+2z+5} dz$, where C is the circle $\lvert z + 1 - i \rvert = 2$.	6