CODE:- 8962

T.Y.B.Sc. EXAMINATION -OCT-2017

M – 302 MATHEMATICAL ANALYSIS

TIME:-3 HOURS

INSTRUCTIONS: (1) All questions are compulsory.
(2) Each question carries equal marks

TOTAL MARKS:-100

QUE. 1 A For 0 < a<b ,prove that $\frac{\pi^3}{24} \le \int_0^{\pi} \frac{x^2}{5+3\cos x} dx \le \frac{\pi^3}{6}$ 7M

B Prove : If f ,g ∈ $R[a,b]$then f+ g ∈ $R[a,b]$ and $\int_a^b [f(x) + g(x)]\,dx = \int_a^b f(x)\,dx + \int_a^b g(x)\,dx$ 7M

C State and prove first mean value theorem for integral calculus. 6M

OR

QUE. 1 A State and prove Fundamental theorem of integral calculus. 7M

B If f ∈ $R[a,b]$ and F(x) = $\int_a^x f(t)dt$, a$\le x \le b$ then F(x) is continuous in [a , b] and if f is continuous in [a , b] then F(x) is derivable and F'(x) = f(x). 7M

C Using definition of R-integration obtain value of $\int_1^2 (3x+1)$dx. 6M

QUE. 2 A (X , d) is discrete metric space. prove that N(a, δ) = { a } , 0 < $\delta \le 1$ and N(a, δ) = X ,δ > 1. 7M

B X≠ ∅,d : X x X →R ,(1) d(x , y) = 0 ⇔ x = y
(2) d(x, y) ≤ d(x , z) + d(y ,z) ∀ $x, y, z \in X$ prove that (X , d) is metric space. 7M

C Prove: Finite intersection of open sub sets of metric space is open. 6M

OR

QUE. 2 A State and prove Housedorff's property for metric space. 7M

B (X , d) is metric space. A ⊂ X , B ⊂ X . prove or dis prove: 7M
(i) $(A \cup B)^\circ = A^\circ \cup B^\circ$ (2) $(A \cap B)^\circ = A^\circ \cap B^\circ$

C Prove : Finite union of closed sub sets of metric space is closed. 6M

QUE. 3 A In usual notation prove that closure of subset of metric space is closed set. 7M

B Prove: cantor set is perfect set. 7M

C Prove: Cauchy sequence is bounded in metric space. 6M

OR

QUE. 3 A Define: convergent sequence in metric space. Prove: In metric space convergent sequence converses to unique limit. 7M

B Show that $\frac{3}{4}$ is in cantor set. 7M

C Prove: (R , d) is complete metric space. 6M

QUE. 4 A If any connected subset of R contains at least two points then it is an interval. 7M

B State and prove abel's test for convergence of improper integrals. 7M

C Discuss convergence of $\int_{-\infty}^{\infty} \frac{dx}{(1+x^2)^2}$ 6M

OR

QUE. 4	A	Prove: Any infinite subset of discrete metric space is not compact.	7M
	B	State and prove Dirichet's test for convergence of improper integrals.	7M
	C	Prove that (1 , 2) is not compact.	6M
QUE. 5	A	Test for uniform convergence of $f_n(x) = \frac{nx}{1+n^3x}, \forall\, x \in [\,0\,,1\,]$.	7M
	B	Prove that set of rational number in [-1 , 1] is countable.	7M
	C	Test for uniform convergence of $\sum \frac{1}{n^4+n^2x^2}$	6M

OR

QUE. 5	A	State and prove waitress's M-test for uniform convergence of series of functions.	7M
	B	Prove : Arbitrary union of countable set is countable and hence set of rational number is countable.	7M
	C	Test for uniform convergence of $f_n(x) = \frac{nx}{1+n^4x}, \forall\, x \in [\,0\,,1\,]$.	6M