T. Y. B. Sc Mathematics Examination March/April-2016
Mathematical analysis
Paper No: M-302

Total Marks: 100

Time : 3 hours
Paper code: 8962

Instructions:

(1) All questions are compulsory.
(2) Each question carry equal marks.

Que-1. (a) State and prove fundamental theorem of integral calculus. [7]

(b) In usual notation prove that $m(b-a) \leq \int_a^b f(x)dx \leq M(b-a)$. [7]

(c) Give a partition P of [0,1]. Find $U(P, f)$ and $L(P, f)$ for $f(x) = x^2$. [6]

or

Que-1. (a) State and prove a necessary and sufficient condition for the integrability of the bounded function f defined on $[a, b]$. [7]

(b) Using the partition $P = \{0, t/n, 2t/n, \ldots, nt/n\}$ of $[0, t]$, show that $\int_0^t \sin x dx = 1 - \cos t$ [7]

(c) Let f be monotonic function on $[a, b]$. Prove that $f \in R[a, b]$. [6]

Que-2. (a) Let (X, d) be a metric space. Show that the function d_1 defined by $d_1(x, y) = \frac{d(x,y)}{1+d(x,y)}$ is a metric on X. [7]

(b) In usual notation show that $\text{int}(A \cup B) = \text{int(A)} \cup \text{int(B)}$. [7]

(c) Prove or disprove: Arbitrary intersection of open sets is an open set in any metric space. [6]

or

Que-2. (a) Prove that any subset of a discrete metric space is open as well as closed. [7]

(b) Define *interior* of a set. Show that a subset A of a metric space X is open if and only if $A = \text{int}(A)$. [7]

(c) Let $(\mathbb{R}, d)$ be the usual metric space. Find interior of the set $A = [3, 4] \cap \{\pi\}$. [6]

Que-3. (a) Define *Cantor set.* Show that Cantor set is perfect. [7]

(b) Prove that continuous image of compact set is compact. [7]

(c) Define *dense set.* Show that $\mathbb{Q}$ is dense in the usual metric space $(\mathbb{R}, d)$. [6]

or

Que-3. (a) Define *closure* of a set. In usual notation prove that $\overline{A \cup B} = \overline{A} \cup \overline{B}$. [7]

(b) Show that every convergent sequence is Cauchy sequence in any metric space. [7]

(c) Define *boundary* of a set. Find boundary of the set $(1, 2] \cup (3, 4)$ in the usual metric space $(\mathbb{R}, d)$. [6]

Que-4. (a) Define *connected set.* Show that continuous image of connected set is connected. [7]

(b) State and prove Abel's test. [7]

(c) Show that $\int_0^\infty \frac{x}{x^3+1} dx$ is convergent. [6]

or

Que-4. (a) State and Prove Dirichlet's test. [7]

(b) Show that every absolutely convergent integral is convergent. [7]

(c) Test the convergence of the improper integral $\int_0^\infty \frac{x\tan^{-1}x}{(1+x^4)^{1/3}}\,dx$. [6]

Que-5. (a) Test the uniform convergence of the sequence $\{f_n\}$, where $f_n(x) = \frac{nx}{1+n^2x^2}$, $x \in \mathbb{R}$. [7]

(b) Test the uniform convergence of the series $\sum\limits_{n=1}^{\infty} \frac{2^n x^{2n-1}}{1+2x^{2n}}$. [7]

(c) Prove that $\mathbb{Z}$ is a countable set. [6]

or

Que-5. (a) Define *uncountable set.* Show that $\mathbb{R}$ is an uncountable set. [7]

(b) Show that the sequence $\{f_n\}$, where $f_n(x) = \tan^{-1} nx$, $x \geq 0$ is uniformly convergent in $[1, 2]$. [7]

(c) Prove that the series $\sum\limits_{n=1}^{\infty} (-1)^n \frac{x^2+n}{n^2}$ converges uniformly on $[1, 4]$. [6]