CODE:8968

T. Y. B. Sc. Exam,' March 2017

Statistics Paper - 302

Time : 3 Hours — Marks : 75

Instructions:- 1) There are five compulsory questions in this Q. Paper.

2)all question carry equal marks.

3) Statistical Tables will be provided on request.

Q1 a) Name the properties to be possessed by an estimator to be good estimator , according to Prof. R. A. Fisher. Explain the sufficiency. 8

b) Let x_1, x_2, x_3 be a random sample from a poisson distribution with mean θ.Find the efficiency of $\frac{3x_1+2x_2+x_3}{6}$ relative to $\overline{X}$. 7

OR

Q1 a) Stating regularity conditions prove C.R. Inequality. 10

b) Define Most efficient estimator, Minimum Variance Unbiased Estimator. Show that, if M.V.U.E exists then it is unique. 5

Q2 a) State different methods available for finding estimators. Explain the method of moments in detail. 8

b) If $x_1, x_2, \ldots, x_n$ is a random sample of size n from the probability mass function 7

$$p(\theta) = \frac{e^{-\theta}}{x!}\theta^{x} \quad ,x = 0,1,2,\ldots\ldots; \theta > 1$$
$$= 0, \quad \text{elsewhere}$$

Estimate θ by the method of maximum likelihood and the method of moments.

OR

Q2 a) Justify the importance of a consistent estimator in Statistical Inference. 5

b) If $x_1, x_2, \ldots\ldots\ldots x_n$ is a r.s. from a $N(\mu, 16)$, Estimate μ the method of maximum likelihood.Do you agree that the obtained MLE is sufficient estimator of μ? 10

Q.3 (a) Describe, in detail, about a confidence interval. Also, state its importance. 8

(b) If $x_1, x_2, \ldots\ldots\ldots x_{25}$ is an observed r.s. of size 25 from a $N(\mu,16)$ with sample total=200,find an 95% confidence interval for μ. 7

OR

Q3 a) Explain, in brief, Decision Theory. With reference to decision theory, 9 define terms: i) Decision Rule ii) Risk Function iii) Maximax Decision Rule

b) Use the minimax criterion to estimate a parameter θ of a binomial distribution on the basis of the random variable *X*, the observed number of successes in n trials, when the decision function is of the form *d(x)* = *(x+a) / (n + b)*, and the loss function is $L\left(\frac{x+a}{n+b},\theta\right)=c\left(\frac{x+a}{n+b}-\theta\right)^2$ 6

Where a and b are constants and c is a positive constant. Then, determine values of a and b.

Q4 a) What is testing of hypothesis? State its importance in statistical inference. Hence or otherwise, define null and alternative hypotheses, critical region and Type I and type II errors. 8

b) If $x_1,x_2,\ldots\ldots\ldots x_{16}$ is an observed r. s. from a $N(\mu,\sigma^2=64)$ and it is desired to test $H_o:\mu = 60$, against H_1: $\mu = 65$, then Obtain a best C.R. to test H_0 against H_1 and its power function. (Use 5% level of significance). 7

OR

Q4 a) Define the following Terms- 3

i) Most Powerful Test,

ii) Uniformly Most Powerful Test,

iii) Best Critical Region.

b) State and prove the Neyman-Pearson Theorem to find a best critical region of level of significance α. 6

c) If a random variable X has poisson distribution with parameter μ, then derive an UMP Critical region to test H_o: $\mu=\mu_0$ vs H_1: $\mu = \mu_1$, $(\mu_0<\mu_1)$. 6

Q5 a) Define a general Linear model. Explain a few situations in which it can be used. 6

b) What is full rank general linear model? 9

In usual notations, prove following:

$\underline{\hat{\beta}} = (X'X)^{-1}X'\underline{Y}$ and (2) $E(\underline{\hat{\beta}}) = \underline{\beta}$

OR

Q5 a) State and prove the Gauss- Markoff theorem. Mention its utility. 8

b) For a linear model $\left(\underline{Y}, X\underline{\beta}, \sigma^2 I\right)$, prove that any linear parametric function $\underline{C}'\underline{\beta}$ is estimable, if and only if rank $(X' \ \underline{C})$ = rank X'. 7

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