Code: 21501      B.Sc. Semester – 5 CBCS (NEW)

MAT-CC-506: Complex Analysis-I

Time: 2 ½ Hours      Total Marks: 70

Note: Notations used are standard notations.

Q.1 (a) State and prove the De – Moiver's theorem for rational number. [9]

(b) Prove that $x + \frac{1}{x} = 2\cos\theta$ for $x = e^{i\theta}$ [5]

OR

Q.1 (a) If $x + \frac{1}{x} = 2\cos\alpha$ and $y + \frac{1}{y} = 2\cos\beta$ then prove that [7]

$$(i) x^m y^n + \frac{1}{x^m y^n} = 2\cos(m\alpha + n\beta)$$

$$(ii) x^m y^n - \frac{1}{x^m y^n} = 2i\sin(m\alpha + n\beta)$$

(b) Find the roots of : $(i)\ x^9 + x^5 + x^4 + 1 = 0$ $(ii)\ x^6 + i = 0$ [7]

Q.2 (a) Obtain expansion of $(i)\ \sin 7\theta$ $(ii)\ \cos 7\theta$ [7]

(b) Compute the expansion of $\sin^8\theta$ in terms of $\cos\theta$ only. [7]

OR

Q.2 (a) Expand $\tan\alpha$ in terms of $\alpha$. [7]

(b) Compute the expansion of $\sin^7\theta$ in terms of $\sin\theta$ only. [7]

Q.3 (a) Find the value : $(i)\ e^{2+3\pi i}$ $(ii)\ e^{\frac{1}{2}+\frac{\pi}{4}i}$ $(iii)\ e^{2+\pi i}$ [7]

(b) Obtain the expansion series of $\cos hY$ and $\sin hY$ in terms of $Y$. [7]

OR

Q.3 (a) Derive relation between circular and hyperbolic functions. [7]

(b) If $\tan\frac{x}{2} = \tan h\frac{u}{2}$ then prove that $(i)\ \sin h\,u = \tan x$ $(ii)\ \cos h\,u = \sec x$ [7]

Q.4 (a) Prove that $\tanh^{-1}\left(\frac{x-y}{x+y}\right) = \frac{1}{2}\log\left(\frac{x}{y}\right)$ [7]

(b) Solve: $(i)\ \sin h\,z = i$ $(ii)\ \cos h\,z = \frac{1}{2}$ $(iii)\ \cos z = 2$ [7]

1

$$(iv) \exp(2z - 1) = 1 \quad (v) \exp(z) = -2.$$

<center>OR</center>

Q.4 (a) Define the inverse of : $(i)$ sine function, $(ii)$ cosine function and $(iii)$ [7]
Tangent function.

(b) Prove that $\log\left(\frac{a+ib}{a-ib}\right) = 2i \tan^{-1}\frac{b}{a}$ [7]

Q.5 (a) Obtain $f'(z)$ of : $(i)\frac{1}{z}$ $(ii)$ $\sin z$ $(iii)$ $e^z$ [7]

(b) If a complex function of complex variable is differentiable at point $Z_0$ [7]
then it is continuous at same point but converse is not true.

<center>OR</center>

Q.5 (a) Obtain $f'(z)$ of: $(i)$ $\tan z$ $(ii)$ $\cos^2 z$ $(iii)$ $z^{1/2}$ [7]

(b) Let $f(z) = u + iv, z = x + iy$, $z_0 = x_0 + iy_0$, $w_0 = u_0 + iv_0$, where [7]
$u = u(x,y)$, $v = v(x,y), u_0 = u_0(x_0,y_0)$, $v_0 = v_0(x_0,y_0)$ then show
that $\lim_{z \to z_0} f(z) = w_0 \Leftrightarrow \lim_{(x,y)\to(x_0,y_0)} u(x,y) = u_0$

$$\Leftrightarrow \lim_{(x,y)\to(x_0,y_0)} v(x,y) = v_0$$

<center>----------------</center>

<center>2</center>