

INSTRUCTIONS (1) ALL QUESTIONS ARE COMPULSORY.  
(2) EACH QUESTION CARRY EQUAL MARKS.

- Q.1 A State and prove gram Schmidt orthogonalization process. [7]  
B Apply gram Schmidt process to obtain an orthonormal basis from [7]  
 $\{(0, 0, 2), (2, 1, 0), (-1, 2, 1)\}$ .

OR

- Q.1 A If  $B = \{x_1, x_2, x_3, \dots, x_n\}$  be orthonormal basis and for  $y \in V$  prove that [7]  
 $\|y\|^2 = \sum | \langle y, x_i \rangle |^2$   
B State and prove Schwartz's inequality. [7]  
Q.2 A State and prove riesz representation theorem. [7]  
B Define symmetric linear transformation and verify the following function is linear [7]  
symmetric or not ?  
 $T: R^3 \rightarrow R^3; T(x, y, z) = (2x + 3y + z, 3x + 4y + z, x + y + 5z)$

OR

- Q.2 A  $T: V \rightarrow V$  be a linear function then  $T$  is orthogonal iff  $\|T(X)\| = \|X\|$  [7]  
B If  $W = \text{sp}\{(1, 0, 1), (0, 1, 1)\}$  then obtain  $W^\perp$  [7]  
Q.3 A Prove that  $\begin{vmatrix} a+b & b+c & c+a \\ p+q & q+r & r+p \\ x+y & y+z & z+x \end{vmatrix} = 2 \begin{vmatrix} a & b & c \\ p & q & r \\ x & y & z \end{vmatrix}$  [7]  
B Solve the following linear equation by Cramer's rule. [7]  
 $3x + 4y + 7z = 2; 4y - 7z = 2; 7x + 9y + 63z = 2.$

OR

- Q.3 A Solve the following linear equation by Cramer's rule. [7]  
 $x + 8y = 3; 3x + 4y - 9z = 3; x + 8y + 7z = 3.$   
B Find the value of  $A = \begin{vmatrix} 1 & 1 & 1 & 1 \\ \alpha & \beta & \gamma & \delta \\ y + \beta & y + \delta & \delta + \alpha & \alpha + \beta \\ \delta & \alpha & \beta & \gamma \end{vmatrix}$ . [7]  
Q.4 A If  $T: R^2 \rightarrow R^2, T(x, y) = (x + y, 3x - 2y)$  then find Eigen values and Eigen vectors of  $T$ . [7]  
B Find the direction of principal axis of the conic  $2x^2 + 3xy - 2y^2 = 10$  by diagonalization [7]  
method.

OR

- Q.4 A Verify cayley theorem for the function If  $T: R^3 \rightarrow R^3, T(x, y, z) = (x + z, y - x, x + y + z)$  [7]  
B Find the direction of principal axis of the conic  $x^2 + xy + y^2 = 1$  by diagonalization [7]  
method.  
Q.5 A Define the curve represented by the equation  $2x^2 - 72xy + 23y^2 + 140x - 20y + 50 = 0$ . [7]  
B Verify the distributive law for vectors  $u = 2i - 3j + k$  and  $v = i + j + k$  and  $w = i + 3j + k$  [7]  
for  $u, v, w \in V$

OR

- Q.5 A If  $\phi(x, y) = x_1y_1 - x_2y_1 + x_2y_2$  then  $\phi$  is bilinear or not ? [7]  
B Define the curve represented by the equation [7]  
 $11x^2 + 6xy + 19y^2 = 80$ .