

MARCH-15

CODE:3837

**B.Sc.- SEM-IV EXAMINATION - MARCH**  
M-402: LINEAR ALGEBRA II

TIME :2:30 Hours

TOTAL MARKS:70

INSTRUCTIONS: (1) All questions are compulsory.  
(2) Each question carries equal marks.

Q.1 A State and prove gram- Schmidt orthogonalization process to obtain an orthonormal basis. [07]

B Apply gram- Schmidt process to obtain an orthonormal basis for vector set  $\{(1,1,1,1), (0,2,0,2), (-1,1,3,-1)\}$ . [07]

OR

Q.1 A State and prove Schwartz's inequality. [07]

B Apply gram- Schmidt process to obtain an orthonormal basis for vector set  $\{(1,-1,1,-1), (5,1,1,1), (2,3,4,-1)\}$ . [07]

Q.2 A State and prove riesz representation theorem. [07]

B Find the coordinate of the polynomial  $3 + 4x + 9x^2$  relative to ordered basis  $\{x^2 + 1, x^3\}$ . [07]

OR

Q.2 A T:  $V \rightarrow V$  be a linear function then the following are equivalent. [07]

(1) T is orthogonal. (2)  $\|T(X)\| = \|X\|$ .

(3) If  $\{e_i\}$  for  $i = 1, 2, \dots, n$  is an orthonormal basis then  $\{T(e_i) / i = 1, 2, \dots, n\}$  is an orthonormal basis.

B Define symmetric linear transformation and verify the following functions are linear symmetric or not?  $T: \mathbb{R}^3 \rightarrow \mathbb{R}^3$ ; [07]

$T(x) = (2a_1 + 3a_2 + a_3, 3a_1 + 4a_2 + a_3, a_1 + a_2 + 5a_3)$ ,

$T(x) = \left( \frac{a_1 - a_2}{\sqrt{2}}, \frac{a_1 + a_2 - 2a_3}{\sqrt{6}}, \frac{a_1 + a_2 + a_3}{\sqrt{3}} \right)$ .

Q.3 A Define vector product and prove  $|u \times v| = |u| \cdot |v| \sin \theta$ . [10]

B Prove that, [04]

$$\begin{vmatrix} 1+a & 1 & 1 & 1 \\ 1 & 1+b & 1 & 1 \\ 1 & 1 & 1+c & 1 \\ 1 & 1 & 1 & 1+d \end{vmatrix} = (abcd) \left( 1 + \frac{1}{a} + \frac{1}{b} + \frac{1}{c} + \frac{1}{d} \right)$$

OR

Q.3 A Find the value of  $A = \begin{vmatrix} 1 & 1 & 1 & 1 \\ \alpha & \beta & \gamma & \delta \\ \gamma + \beta & \gamma + \delta & \delta + \alpha & \alpha + \beta \\ \delta & \alpha & \beta & \gamma \end{vmatrix}$ . [10]

B Solve the following linear equation by Cramer's rule. [04]

$$3x + 4y + 7z = -1 ; 4y - 7z = 52 ; 7x + 9y + 63z = 32.$$

Q.4 A Verify  $\begin{vmatrix} x & y & z \\ x^2 & y^2 & z^2 \\ yz & zx & xy \end{vmatrix} = \begin{vmatrix} 1 & 1 & 1 \\ x^2 & y^2 & z^2 \\ x^3 & y^3 & z^3 \end{vmatrix}$ . [07]

B Find the direction of principal axis of the conic  $8x^2 - 12xy + 17y^2 = 80$  by diagonalization method. [07]

OR

- Q.4 A If  $T: \mathbb{R}^3 \rightarrow \mathbb{R}^3$ ,  $T(x, y, z) = (x + y + 2z, y, x - 2y + z)$  then find Eigen values and Eigen vectors of T. [07]
- B Find the direction of principal axis of the conic  $2x^2 + 3xy - 2y^2 = 10$  by diagonalization method. [07]
- Q.5 A Discuss classification of quadrics. [07]
- B Identify the curve represented by the equation  $2x^2 - 72xy + 23y^2 + 140x - 20y + 50 = 0$ . [07]
- OR
- Q.5 A State and prove the properties of bilinear form on a vector space V. [07]
- B If  $\phi(x, y) = x_1y_1 - x_1y_2 + 2x_2y_1 + 5x_2y_2$  then  $\phi$  is bilinear or not ? [07]