

INSTRUCTIONS(1)ALL QUESTIONS ARE COMPULSORY.  
(2)EACH QUESTION CARRY EQUAL MARKS.

- Q.1 A State and prove Schwartz's inequality. [7]  
B Apply gram- Schmidt process to obtain an orthonormal basis for  $\{(1,0,1), (1,1,0), (0,1,1)\}$ . [7]  
OR
- Q.1 A State and prove parallelogram law. [7]  
B State and prove gram Schmidt orthogonalization process. [7]
- Q.2 A If T is symmetric linear transformation if  $R_T$  is range of T, and  $N_T$  is nullity of T then  $R_T = (N_T)^\perp$  [7]  
B If W is subspace of an inner product space V then prove that,  $\text{Dim } W^\perp = \text{Dim } V - \text{Dim } W$  [7]  
OR
- Q.2 A Define symmetric linear transformation and verify the following function is linear symmetric or not?  $T: \mathbb{R}^2 \rightarrow \mathbb{R}^2$ ;  $T(x,y) = \left( \frac{x+\sqrt{3}y}{2}, \frac{y-\sqrt{3}x}{2} \right)$  [7]  
B State and prove riesz representation theorem. [7]
- Q.3 A Prove that  $\begin{vmatrix} x & y & z \\ x^2 & y^2 & z^2 \\ yz & zx & xy \end{vmatrix} = \begin{vmatrix} 1 & 1 & 1 \\ x^2 & y^2 & z^2 \\ x^3 & y^3 & z^3 \end{vmatrix}$  [7]
- B Verify the matrix  $\begin{bmatrix} 1 & -2 & -2 \\ 2 & -1 & 2 \\ 2 & 2 & -1 \end{bmatrix}$  is orthogonal or not? [7]  
OR
- Q.3 A Find the value of  $A = \begin{vmatrix} 1 & 1 & 1 & 1 \\ \alpha & \beta & \gamma & \delta \\ y + \beta & y + \delta & \delta + \alpha & \alpha + \beta \\ \delta & \alpha & \beta & \gamma \end{vmatrix}$ . [7]
- B Verify  $T: \mathbb{R}^3 \rightarrow \mathbb{R}^3$ ,  $T(a,b,c) = (a+b, a+2b+c, b+c)$  is symmetric linear or not? [7]
- Q.4 A Define the properties of vector product and prove that  $|u \times v| = |u|^2 |v|^2 - |u \cdot v|^2$  [7]  
B V be a vector space f, g be linear functional on V,  $h: V \rightarrow F$  is defined as  $h(x \cdot y) = f(x) \cdot g(y)$  then show that h is bilinear form. [7]  
OR
- Q.4 A Define bilinear form and if  $\phi_1$  and  $\phi_2$  are bilinear form on vector space V then prove that  $\phi_1 + \phi_2$  is also bilinear form. [7]  
B Classify the quadratic curve represented by the equation,  $2x_1^2 - 72x_1x_2 + 23x_2^2 + 14x_1 - 20x_2 + 50 = 0$  [7]
- Q.5 A Verify cayley theorem for the function If  $T: \mathbb{R}^2 \rightarrow \mathbb{R}^2$ ,  $T(x,y) = (x+2y, 3x+2y)$ . [7]  
B If  $\Phi$  is bilinear form then prove that  $\alpha\Phi$  is bilinear form [7]  
OR
- Q.5 A Define the curve represented by the equation  $x^2 + z^2 + 6yz - 2xz + 4xy$ . [7]  
B  $B: \mathbb{R}^n \times \mathbb{R}^n \rightarrow \mathbb{R}$  is symmetric linear form if  $Q(x) = B(x, x)$  then show that  $Q(X+Y) + Q(X-Y) = 2[Q(X) + Q(Y)]$ . [7]