Code : 4312 SEM-V EXAMINATION,MARCH APRIL -2015

M-501: ABSTRACT ALGEBRA

TIME:2:30 Hours TOTAL MARKS:70

INSTRUCTIONS: (1) All questions are compulsory.
(2) Each question carries equal marks.

Q.1 A $G = R-\{1\}$,$*$ is defined on set G, $a*b = a+b-ab$. show that $(G,*)$is a group and solve the equation : $2^{-1}*3^{-1}*x = 5^{-1}$ in G. [7]

B (G , $*$) is a group. For a ,b,c$\in$ G, (1) $(a*b)^{-1} = a^{-1} * b^{-1}$. [7]
(2) $a*b = a*c \Rightarrow b=c$

OR

Q.1 A If G is a group, then for all a,b $\in$G, the equations :ax=b and ya=b have unique solution in G. [7]

B Prove that the set G= {0,1,2,3,4,5,6} is finite abelian group of order 7with respect to addition modulo 7. [7]

Q.2 A State and prove Lagrange's theorem [7]

B If H and K are two subgroups of a group G then HK is subgroup of G iff HK=KH [7]

OR

Q.2 A The order of the elements 'a' and $x^{-1}ax$ are the same where a ,x are any two elements of group. [7]

B If H and K are finite subgroups of G then prove that $O(HK) = \frac{O(H)O(K)}{O(H\cap K)}$ [7]

Q.3 A Prove that the set P_n of all permutations define on n symbols is group of order n! [7]

B $f = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 3 & 1 & 4 & 5 & 6 & 2 \end{pmatrix}$; $g = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 2 & 4 & 1 & 3 & 6 & 5 \end{pmatrix}$; $h = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 5 & 4 & 2 & 3 & 1 & 6 \end{pmatrix}$ then find [7]
$f^2, g^3, fg^2, h^{-1}, fgf^{-1}, ghg^{-1}$ also verify it is even permutation or odd permutation

OR

Q.3 A If G is group and a$\in$ G.H = { x$\in$ G/ ax = xa } then H $\le$ G. [7]

B Obtain Alternative subgroup A_3 of group S_3 . [7]

Q.4 A State and prove cayle's theorem. [7]

B If $\emptyset$: G$\to$ $G'$is an isomorphim then for any a $\in$ G then prove that [7]
$o(a) = O\,[\emptyset(a)]$

OR

Q.4 A $\emptyset : G \to G'$ is homomorphism of group then prove that $\emptyset$ is one –one iff $\ker\emptyset = \{e\}$ [7]

B Prove that two cyclic group having same order are isomorphic. [7]

Q.5 A State and prove fundamental theorem of homomorphism. [7]

B If H and K are normal subgroup of group G then prove that ; [7]
(1) H$\cap$K is normal subgroup in group G.
(2) H$\cap$K = {e} then hk = kh.

OR

Q.5 A Let G is group and G' is commutate subgroup of G then , [7]
(1) G' is normal in G.
(2) G/G' is abelian.
If N is any normal subgroup of G then G/N is abelian iff $G' \subseteq$ N.

B If F is homomorphism from G intoG' with kernel K then K is normal sub group of G. [7]