- 2 APR 2019

Code: 21501 **B.Sc. Semester – 5 CBCS (NEW)**

MAT-CC-506: Complex Analysis-I

Time: 2 ½ Hours **Total Marks: 70**

Note: Notations used are standard notations.

Q.1 (a) State and prove the De – Moiver's theorem for rational number. [9]

(b) Prove that $x+\frac{1}{x}=2\cos\theta$ for $x=e^{i\theta}$ [5]

OR

Q.1 (a) If $x+\frac{1}{x}=2\cos\alpha$ and $y+\frac{1}{y}=2\cos\beta$ then prove that [7]

$$(i) x^m y^n+\frac{1}{x^m y^n}=2\cos(m\alpha+n\beta)$$

$$(ii) x^m y^n-\frac{1}{x^m y^n}=2i\sin(m\alpha+n\beta)$$

(b) Find the roots of : $(i)\ x^9+x^5+x^4+1=0$ $(ii)\ x^6+i=0$ [7]

Q.2 (a) Obtain expansion of $(i)\sin 7\theta$ $(ii)\cos 7\theta$ [7]

(b) Compute the expansion of $\sin^8\theta$ in terms of $\cos\theta$ only. [7]

OR

Q.2 (a) Expand $\tan\alpha$ in terms of α. [7]

(b) Compute the expansion of $\sin^7\theta$ in terms of $\sin\theta$ only. [7]

Q.3 (a) Find the value : $(i)\ e^{2+3\pi i}$ $(ii)\ e^{\frac{1}{2}+\frac{\pi}{4}i}$ $(iii)\ e^{2+\pi i}$ [7]

(b) Obtain the expansion series of $\cos hY$ and $\sin hY$ in terms of Y. [7]

OR

Q.3 (a) Derive relation between circular and hyperbolic functions. [7]

(b) If $\tan\frac{x}{2}=\tan h\frac{u}{2}$ then prove that $(i)\sin h\,u=\tan x$ $(ii)\cos h\,u=\sec x$ [7]

Q.4 (a) Prove that $\tanh^{-1}\left(\frac{x-y}{x+y}\right)=\frac{1}{2}\log\left(\frac{x}{y}\right)$ [7]

(b) Solve: $(i)\sin h\,z=i$ $(ii)\cos h\,z=\frac{1}{2}$ $(iii)\cos z=2$ [7]

$(iv)\ \exp(2z-1) = 1 (v)\ \exp(z) = -2.$

OR

Q.4 (a) Define the inverse of : (i) sine function, (ii) cosine function and (iii) Tangent function. [7]

(b) Prove that $\log\left(\frac{a+ib}{a-ib}\right) = 2i\tan^{-1}\frac{b}{a}$ [7]

Q.5 (a) Obtain $f'(z)$ of : $(i)\ \frac{1}{z}$ $(ii)\ \sin z$ $(iii)\ e^z$ [7]

(b) If a complex function of complex variable is differentiable at point Z_0 then it is continuous at same point but converse is not true. [7]

OR

Q.5 (a) Obtain $f'(z)$ of: $(i) \tan z\ (ii) \cos^2 z\ (iii)\ z^{1/2}$ [7]

(b) Let $f(z) = u + iv, z = x + iy,\ z_0 = x_0 + iy_0,\ w_0 = u_0 + iv_0,$ where $u = u(x,y),\ v = v(x,y), u_0 = u_0(x_0, y_0),\ v_0 = v_0(x_0, y_0)$ then show that $\lim_{z\to z_0} f(z) = w_0 \Leftrightarrow \lim_{(x,y)\to(x_0,y_0)} u(x,y) = u_0$ [7]

$\Leftrightarrow \lim_{(x,y)\to(x_0,y_0)} v(x,y) = v_0$
