

Code :4316

SEM-V EXAMINATION, ~~Nov~~ 2014  
M-505: COMPLEX ANALYSIS-I

TIME:2:30 HOURS

TOTAL  
MARKS:70

INSTRUCTIONS: (1) All questions are compulsory.  
(2) Each question carries equal marks.

- Q.1 A State & prove De' moivre's theorem [10]  
B Expand :  $\cos^n \theta$  in terms of  $\cos \theta$  [4]
- OR
- Q.1 A  $a = \text{Cis}2\alpha, b = \text{Cis}2\beta, c = \text{Cis}2\gamma, d = \text{Cis}2\delta$  then prove that , [8]  
(i)  $\sqrt{abcd} + \frac{1}{\sqrt{abcd}} = 2\cos(\alpha + \beta + \gamma + \delta)$   
(ii)  $\frac{\sqrt{ab}}{\sqrt{cd}} + \frac{\sqrt{ad}}{\sqrt{bc}} = 2\cos(\alpha + \beta - \gamma - \delta)$
- B  $z_1 = 1 + 2i, z_2 = 2 + i \Rightarrow \arg\left(\frac{z_1}{z_2}\right) = \arg z_1 - \arg z_2$  [6]
- Q.2 A If  $x = \text{cis}\alpha, y = \text{cis}\beta, z = \text{cis}\gamma, x+y+z = 0 \Rightarrow \frac{1}{x} + \frac{1}{y} + \frac{1}{z} = 0$  [7]  
B Prove :  $1 + \cos 14\theta = 2(64\cos^7\theta - 112\cos^5\theta + 56\cos^3\theta - 7\cos\theta)$  [7]
- OR
- Q.2 A Prove :  $(a+ib)^{m/n} + (a-ib)^{m/n} = 2(a^2+b^2)^{m/2n} \cos\left(\frac{m}{n}\tan^{-1}\frac{b}{a}\right)$  [7]  
B Expand :  $\cos\alpha$  in terms of  $\alpha$  [7]
- Q.3 A find value of  $(1+i\sqrt{3})^8 + (1-i\sqrt{3})^8$  [7]  
B prove that  $\cos 8\theta = 1 - 32\sin^2\theta + 160\sin^4\theta - 256\sin^6\theta + 128\sin^8\theta$  [7]
- OR
- Q.3 A Prove that: (1)  $(\sin(A+B) - e^{Ai}\sin B)^n = \sin^n A e^{-ni\theta}$  [7]  
(2)  $\cosh^6 y - \sinh^6 y = 1 + \frac{3}{4}\sinh^2 y$  (3)  $\cosh(x+y) \cdot \cosh(x-y) = \cosh^2 y + \sinh^2 y$
- B Prove that :  $\left(\frac{1+\tanh 2A}{1-\tanh 2A}\right)^3 = \sinh 6A + \cosh 6A$  [7]
- Q.4 A  $\tanh \alpha = \tan \alpha \tanh \beta, \tanh z = \cot \alpha \tanh \beta \Rightarrow \tan(y+z) = \sinh 2\beta \operatorname{cosec} 2\alpha$  [7]  
B Prove :  $\frac{1-\cosh x}{1+\cosh x} = -(\operatorname{cosech} x - \coth x)$  [7]
- OR
- Q.4 A Solve:  $\sinh z = i$  [7]  
B Prove  $\tanh^{-1}\left(\frac{2x}{1+x^2}\right) = \sinh^{-1}\left(\frac{2x}{1-x^2}\right)$  [7]
- Q.5 A Obtain bilinear transformations from  $z$ -plane onto  $\omega$  - plane [7]  
1)  $z_1=0, z_2=1, z_3=-1$  onto  $\omega_1=1, \omega_2=i, \omega_3=\infty$
- B show that the set of all bilinear mapping is a group under composition [7]
- OR
- Q.5 A show that composition of two mobius mappings is again mobius mapping [7]  
B Prove :  $\operatorname{Log}\left(\frac{1}{1-e^{ix}}\right) = \log\left(\frac{1}{2}\operatorname{cosec}\frac{x}{2}\right) + i\left(\frac{\pi}{4} - \frac{\theta}{2}\right)$  [7]