Code : 4316

B.Sc.SEM-V EXAMINATION - Oct-2015

M-505: COMPLEX ANALYSIS-I

TIME :2:30 HOURS

TOTAL MARKS:70

INSTRUCTIONS: (1) All questions are compulsory.
(2) Each question carries equal marks.

Q.1 A State & prove De' moivre's theorem [7]

B Prove the identities: (1) $1-\tanh^2 y=\operatorname{sech}^2 y$ (2) $\cosh^2 y-\sinh^2 y=1$ [7]

(3) $\cosh(z_1+z_2) = \cosh z_1 \cdot \cosh z_2 + \sinh z_1 \cdot \sinh z_2$

OR

Q.1 A $x=\text{Cis}\alpha, y=\text{Cis}\beta$ and then prove that [7]

(1) $\frac{x-y}{x+y} = i\tan\frac{(\alpha-\beta)}{2}$ (2) $\frac{(x+y)(xy-1)}{(x-y)(xy+1)} = \frac{\sin\alpha+\sin\beta}{\sin\alpha-\sin\beta}$

B $a=\text{Cis}2\alpha$, $b=\text{Cis}2\beta$, $c=\text{Cis}2\gamma$, $d = \text{Cis}2\delta$ then prove that [7]

(i) $\sqrt{abcd} + \frac{1}{\sqrt{abcd}} = 2\cos(\alpha+\beta+\gamma+\delta)$ (ii) $\frac{\sqrt{ab}}{\sqrt{cd}} + \frac{\sqrt{ad}}{\sqrt{bc}} = 2\cos(\alpha+\beta-\gamma-\delta)$

Q.2 A Prove : $\frac{1}{(1-\sin\alpha-i\cos\alpha)} = \frac{1}{8}\sec^3\left(\frac{\pi}{4}+\frac{\alpha}{2}\right)\text{cis}\left(\frac{3\pi+6\alpha}{4}\right)$ [7]

B $x+\frac{1}{x} = 2\cos A$ and $y+\frac{1}{y} = 2\cos B \Rightarrow x^m y^n + \frac{1}{x^m y^n} = 2\cos(mA+nB)$ [7]

OR

Q.2 A Expand : $\cos\alpha$ in terms of α [7]

B find value of $(1+i\sqrt{3})^8+(1-i\sqrt{3})^8$ [7]

Q.3 A Prove that $(a+ib)^{m/n}+(a-ib)^{m/n} = 2\,(a^2+b^2)^{m/2n}\cos(\frac{m}{n}\tan^{-1}\frac{b}{a})$ [7]

B Write $|\exp(2z+i)|$ and $|\exp(iz^2)|$ in terms of x and y then prove [7]

$|\exp(2z+i)+\exp(iz^2)| \le e^{2x}+e^{-2xy}$

OR

Q.3 A $x=\text{Cis}\alpha, y=\text{Cis}\beta$ and then prove that [7]

(1) $\frac{x-y}{x+y} = i\tan\frac{(\alpha-\beta)}{2}$ (2) $\frac{(x+y)(xy-1)}{(x-y)(xy+1)} = \frac{\sin\alpha+\sin\beta}{\sin\alpha-\sin\beta}$

B Expand cosh y in terms of y [7]

Q.4 A $\log\cos(x+iy) = \frac{1}{2}\log\frac{\cosh 2y+\cos 2x}{2} - i\tan^{-1}(\tan x \cdot \tanh y)$ [7]

B $\tan(A+iB)=x+iy \Rightarrow x^2+y^2+2x\cdot\cot 2A = 1$ and $x^2+y^2-2y\cdot\coth 2B+1=0$ [7]

OR

Q.4 A $\log\cos(\theta-\emptyset i) = A+Bi$ [7]

$\Rightarrow A=\frac{1}{2}\log\left(\frac{\cos 2\theta+\cosh 2\emptyset}{2}\right)$ and $\emptyset = \frac{1}{2}\log\left(\frac{\sin(\theta+B)}{\sin(\theta-B)}\right)$

B $\tan(\theta - Ai) = \tan\alpha + i\sec\alpha \Rightarrow e^{2A} = \pm\cot\frac{\alpha}{2}$ and $2\theta = n\pi+\frac{\pi}{2}+\alpha$ [7]

Q.5 A show that composition of two mobius mappings is again mobius mapping [7]

B Obtain bilinear transformations from z-plane onto $\omega - plane$ [7]

$z_1=1$, $z_2=\infty$, $z_3=-1$ onto $\omega_1=3$, $\omega_2=0$, $\omega_3=-3$

OR

Q.5 A Prove : $\text{Log}\left(\frac{1}{1-e^{ix}}\right) = \log\left(\frac{1}{2}\text{cosec}\frac{x}{2}\right) + \text{I}\left(\frac{\pi}{4}-\frac{\theta}{2}\right)$ [7]

B find Critical points of the bilinear mappings [7]

(1) $\omega = \frac{az+bz}{cz+d}$, $ad-bc\neq 0$ (2) $\omega = \frac{3z}{1-2z}$