

CODE : 4313

B.Sc.EXAMINATION, March/ April

SEMESTER -V

PAPER NO:M-502

MATHEMATICAL ANALYSIS-I

March 2015

TIME:2:30 HOURS

TOTAL MARKS:70

INSTRUCTIONS: (1) All questions are compulsory.
(2) Each question carries equal marks.

- Q.1 A If $f \in R[a, b]$ and $g \in R[a, b]$, then prove that $f + g \in R[a, b]$. 07
B Every monotonic function is R-integrable over $[a, b]$ 07
OR
- Q.1 A Prove that $\frac{\pi^3}{51} \leq \int_0^\pi \frac{x^2}{10+7\cos x} dx \leq \frac{\pi^3}{9}$ 07
B Find $L(P, f)$ and $U(P, f)$ for function $f(x) = \frac{10}{x}$, $x \in [2, 20]$ with partition $P = \{2, 4, 5, 20\}$ 07
- Q.2 A State and prove condition of R- integrability. 07
B State and prove second mean value theorem. 07
OR
- Q.2 A Prove that neighbourhood of a point in a Metric space is a open set. 07
B $f: [0, 1] \rightarrow R$, $f(x) = x^2$, then prove that $f \in R[0, 1]$ and find $\int_0^1 x^2 dx$ 07
- Q.3 A If (X, d) is a discrete metric space, then prove that 07
 $0 < \delta \leq 1 \Rightarrow N(a, \delta) = \{a\}$ and $\delta > 1 \Rightarrow N(a, \delta) = X$.
B State and prove Hausdroff's property for metric space. 07
OR
- Q.3 A Prove that closure of any subset of metric space is closed. 07
B If (X, d) is metric space, then prove that $(X, \frac{d}{1+d})$ is also metric space. 07
- Q.4 A Define pseudo metric space and give an example of metric space which is pseudo metric space but not metric space. 07
B Prove that sum of two metric spaces is also metric space. 07
OR
- Q.4 A Prove or disprove: (i) $\overline{A \cup B} = \overline{A} \cup \overline{B}$. 07
(ii) $\overline{A \cap B} = \overline{A} \cap \overline{B}$.
B (X, d) is metric space. $E \subset X$ is open if and only if $X - E$ is closed. 07
- Q.5 A Prove that every open sphere is open set. 07
B Define: boundary point, isolated point, dense set, perfect set 07
Cauchy sequence, convergent sequence, closure.
OR
- Q.5 A Prove that every convergent sequence is Cauchy, but converse is not true. 07
B Define: isolated point, Boundary point, Dense set and prove that set of rational number is dense in R. 07