B. SC. SEM –V EXAMINATION OCT/NOV 2015

MATHEMATICS | **PAPER NO. M-501** | **ABSTRACT ALGEBRA**

4312

TIME :2 :30 HOURS TOTAL MARKS:70

INSTRUCTIONS: (1) All questions are compulsory.
(2)Each question carries equal marks.

Que 1. (a) (8)
$G = Q-\{0\}$ a ,b∈ G , a ∗ b = ab/4 prove that (G , ∗) is commutative group.Also solve equation $2 * 3^{-1} * x=5$ in G.

(b) In usual notation prove that $O(a) = O(a^{-1})$ (6)

OR

Que 1. (a) (G , ∗) is a group. For a ,b∈ G, a ∗ x = b and y ∗ a = b has unique solution. (7)

(b) G =R-{1 } a ,b∈ G , a ∗ b = a + b - ab prove that (G , ∗) is commutative group.Also solve equation $5 * 2^{-1} * x=2015$ in G. (7)

Que 2. (a) State and prove Lagrange's theorem. (7)

(b) Prove that the set G={1,2,3,4,5,6} is finite abelian group of order 6 with respect to multiplication modulo 7. (7)

OR

Que 2. (a) prove that H is subgroup of group G iff $ab^{-1} \in H$ for all a , b ∈ H. (7)

(b) The order of the elements 'a' and $x^{-1}ax$ are the same where a ,x are any two elements of group. (7)

Que 3. (a) If G is group and a ∈ G.H = { x∈ G/ ax = xa } then H ≤ G. (7)

(b) Define cyclic group. Prove that (Z_6 ,$+_6$) is cyclic group. (7)

OR

Que 3. (a) If $f = \begin{pmatrix} 1\,2\,3\,4\,5\,6\,7 \\ 7\,3\,2\,1\,4\,5\,6 \end{pmatrix}$; $g = \begin{pmatrix} 1\,2\,3\,4\,5\,6\,7 \\ 3\,1\,4\,7\,6\,2\,5 \end{pmatrix}$; then find fÒg and gÒf . (7)
Also check $(f\grave{O}g)^{-1} = (g\grave{O}f)^{-1}$ Or not?

(b) Obtain Alternative subgroup A_3 of group S_3 . (7)

Que 4. (a) $\varnothing : G \rightarrow G'$ is homomorphism, ∅ is one-one if and only of kernel ∅ ={e} (7)

(b) G = { 1 , -1 , i ,-i } and G′ = {0 , 1 ,2 ,3} prove that (G , .) and (G′ $+_4$) are isomorphic groups. (7)

OR

Que 4. (a) If H is normal sub group of group G and N is normal sub group of G then H⋂N is normal sub group of G. (7)

(b) Let G be a group and X∈G be any element,f:G→G,f(X)=axa^{-1} for X∈G
Then prove that f is isomorphism. (7)

Que 5. (a) State and prove fundamental theorem of homomorphism. (7)

(b) Isomorphism between two groups is an equivalence relation. (7)

OR

Que 5. (a) $\varnothing : G \rightarrow G'$ is homomorphism prove that ∅ (e) = e′ and
$\varnothing(a^{-1}) = [\varnothing(a)]^{-1}$ for a ∈ G. (7)

(b) Prove that factor group of cyclic group is cyclic. (7)