0 8 DEC 2020

Examination December -2020 Seat No.

B.SC.SEM- V

Mathematics: Paper no. MAT-CC -503 CODE: 21498

GROUP THEORY

Time : 2:00 Hours Total marks - 70

Instruction: All questions are compulsory.

Q-1	A	$(G, *)$ is a group and H is non empty subset of G. $H \leq G$ if and only if $a * b^{-1} \in H, \forall\, a, b \in H$ and using these prove:$H_1$and H_2 are two subgroups of a group G then $H_1 \cap H_2$ is also subgroup of G. Is $H_1 \cup H_2$ is subgroups of a group G ?	17
		OR	
Q-1	A(i)	G is a group, $a \in G$ is foxed element of G and $H = \{x \in G / ax = xa\}$. Prove that $H \leq G$	08
	A(ii)	Prove that fourth root of unity is a commutative group with respect to multiplication.	09
Q-2	A	State and prove Lagrange's theorem and hence in usual notation prove that O(a) / O(G)	17
		OR	
Q-2	A(i)	Prove that the set S_n of all permutations define on n symbols is group of order n!	08
	A(ii)	Show that the n! permutations on n symbols, $\frac{1}{2}$n! are even permutations and $\frac{1}{2}$n! are odd permutations.	09
Q-3	A	State and prove the fundamental theorem of homomorphism group and $(G,\cdot)$ is a group and $a * b = b \cdot a$, prove that $(G, *)$ is a group and $(G, *)$ is an isomorphic to group $(G, \cdot)$	18
		OR	
Q-3	A(i)	$(G, *)$ is.a group and $g \in G$ is foxed element of G.i_g:G→G, $i_g(x) = g * x * g^{-1}\ \forall\, x \in G$.Prove that i_g:G→G is an automorphism.	09
	A(ii)	Prove: A sub group of G cyclic group is cyclic sub group.	09
Q-4	A	State and prove Cayle's theorem.	18
		OR	
Q-4	A(i)	$\emptyset$:G→G' is a homorphism and if $K_\emptyset$ is a kernel of a homorphism then $K_\emptyset$ is a normal subgroup of G .	09
	A(ii)	Prove: Intersection of two normal subgroups of group G is normal subgroup of group G.	09

નોધ:

- પ્રશ્નપત્રનો સમય ૧.૩૦ કલાકનો રહેશે.
- પ્રશ્નપત્રમાં ૪ પ્રશ્નોમાંથી કોઇ પણ ત્રણ પ્રશ્નોના જવાબ આપવાનો રહેશે.
- દરેક પ્રશ્નના ગુણ એકસરખા (૧૪ માર્કના) રહેશે. પ્રશ્નપત્ર કુલ ૪૨ માર્કસ નું રહેશે.