17 OCT 2019

Examination October -2019
B.SC.SEM- V
Mathematics: Paper no. MAT-CC -504
MATHEMATICAL ANALYSIS- I

Seat No.

CODE: 21499

Time : 2:30 Hours

Total marks - 70

Instruction: All questions are compulsory.

Q-1 A If f: [a,b] → R is a bounded function and p , p' $\in p[a,b]$ ∃ p ⊂ p' then prove that . L (p,f) ≤ L (p',f) ≤ U (p',f) ≤ U (p,f) 14

OR

Q-1 A(i) State and prove Darbouxe's theorem. 07

A(ii) Prove that every continuous function is R- integrable. 07

Q-1 B Attempt any three. 03

(i) In Usual notation U(P,f) = ___________ (Fill the blank)

(ii) In usual notation , $\int_a^{\bar{b}} f(x)dx =$ ________ (Fill the blank)

(iii) In usual notation , if p_1 *and* p_2 be two partition of [a,b] then
(a) $L(p_{1},f) \le U(p_2,f)$ (b) $U(p_1,f) \le L(p_{2},f)$ (c) both (a) and (b) (d)None of these

(iv) $f(x) = \frac{20}{x}$, $x \in [2,20]$, partition p = {2,4,5,20} then $\| P \|$ = ________(Fill the blank)

(v) if P = $\{x_0, x_1, x_2, x_3, \ldots, x_n\}$ is partition of [a,b] then
(a) $x_0 \ge x_1 \ge x_2 \ge x_3 \ldots \ge x_n$ (b) $x_0 \le x_1 \le x_2 \le x_3 \ldots \le x_n$
(c) both (a) and (b) (d)All of these

Q-2 A if bounded function f(x) is R-integrable on [a,b] and F(x) = $\int_a^x f(t)dt$, $a \le x \le b$ then 14
(1) F(x) is continuous on [a,b]
(2) If f(x) is continuous on[a,b] then F(x) is differentiable on [a,b] and F'(x) =f(x) , $\forall x \in [a,b]$.

OR

Q-2 A(i) State and prove general form of first mean value theorem of Riemann Integration. 07

A(ii) State and prove fundamental theorem of R- integral. 07

Q-2 B Attempt any three. 03

(i) Write Formula of General Form of Second mean value theorem.

(ii) f,g $\in R_{[a,b]}$ then
(a) f+g $\in R_{[a,b]}$ (b) f - g $\in R_{[a,b]}$ (c) f ·g $\in R_{[a,b]}$ (d) All of these

(iii) In usual Notation , f $\in R_{[a,b]}$ iff For $\varepsilon > 0$
(a) U(p,f) +L(p,f) < ε (b) U(p,f) - L(p,f) < ε (c) L(p,f) - U(p,f) < ε (d) none

(iv) If function f is ________ in [a,b] then f $\in R_{[a,b]}$
(a) Continuous (b) Decreasing function (c) Increasing function (d) All of these

(v) Write Formula of fundamental theorem of integral calculus .

Q-3 A State and prove Hausdroff' s property for metric space and define discrete matric space and obtain it. 14

OR

Q-3 A(i) Prove or disprove : (1) Int (A ∪ B) = Int (A) ∪ Int (B) (2) Int (A ∩ B) = Int (A) ∩ Int (B) 07

A(ii) If (X , d) is matric space ,then prove that (X , $\frac{d}{1+d}$) is also matric space. 07

Q-3 B Attempt any four. 04

(i) If E = (18 , 19) is subset of metric space R then $\bar{E}$ = ….. (Fill the blank)

(ii) If (R,d) is real discrete metric space then N(1.5,0.5) = …..
(A) {1.5} (B) R (C) (1, 2) (D) none of these.

(iii) True or false: a and b are both boundary points of E = [a,b]

(iv) Define: limit point in metric space.

(v) Write any two interior points of E = (2018, 2019).

(vi) True or false: Every closed interval of real line is an open set.

Q-4 A Prove that : every convergent sequence is Cauchy sequence but converse is not true and prove that R is complete metric space. 14

OR

Q-4 A(i) In a metric space, limit of function exists then it is unique. 07

A(ii) Define cantor set and prove that it is closed set. 07

Q-4 B Attempt any four. 04

(i) True or false: Cantor set is perfect set.

(ii) True or false: { n^{2019}/ n∈ N} is convergent sequence.

(iii) True or false: Every convergent sequence converges to unique limit.

(iv) Which one of following is in cantor set.
(A) $\frac{19}{3}$ (B) $\frac{10}{9}$ (C) 1 (D) $\frac{3}{2}$

(v) Define: Convergent sequence.

(vi) { $\frac{1}{\pi n}$ / n∈ N} is/are …….sequence.(convergent /divergent/bounded) (Fill the blank)