Paper – ST- 501
Probability and Distributions
April -2016

Time: $2\frac{1}{2}$ Hours   Marks: 70

Instructions: 1) There are 5 compulsory questions in this question paper.
2) All questions carry equal marks.
3) Use of Scientific calculator is allowed.
4) Statistical tables will be provided on request.

Q1 a) State and prove Chebyshev's Inequality. What it gives us?   9

    b) Let the p.d.f. of a r.v. X is   5

$$f(x)= \frac{1}{2\sqrt{3}} , \quad -\sqrt{3} < X < \sqrt{3}$$

$$= 0 \quad \text{otherwise}$$

Find the actual probability $\{|X-m| \geq \frac{3}{2}\sigma \}$. Compare it with the upperbound obtained by Chebyshev's Inequality for t=1.

OR

Q1 a) Define i) Convergence in Probability,   ii) Convergence almost surely.   6

    b) For a geometric distribution , $P(X) = 2^{-x}$, $x = 1, 2, 3,\ldots\ldots$ 8.

Prove that Chebyshev's Inequality gives Prob. $\{ |x - 2| \leq 2\} > $ ½. While the actual probability is 15/16.

Q2 a) State and prove Weak Law of Large Numbers.   8

    b) Prob.$\{X_k = \pm 1\}$ = 1/k and Prob.$\{X_k = 0\}$ = 1 – 2/k. Check whether   6
        WLLN holds good or not?

OR

Q2 a) i. Define characteristic function. State its properties.   5
        ii. State the Laiponouff's Central Limit Theorem.   2
        iii. State Inversion theorem of Characteristics function.   3

    b) Define   i) Convergence in distribution   5
                 ii) Strong Convergence .

Q3 a) Show that under certain condition to be stated, the hyper geometric   7
        distribution tends to binomial distribution.

    b) Define beta distribution of First kind. Obtain expression for its rth raw   7
        moment. Hence derive its first four raw moments.

OR

Q3 a) Obtain an expression for factorial moment generating function of hyper   8
        geometric distribution. Hence find mean and variance of this
        distribution.

    b) Define Beta distribution of Second kind. Obtain expression for its rth 6
        raw moment. Hence derive its first four raw moments.

Q4 a) Obtain an expression for the marginal p. d. f. of random variable X for a 8
        bi-variate normal distribution.

    b) For the bivariate normal distribution with joint p.d.f. 6

$$f(x, y) = k\, e^{-8/27}[(x - 7)^2 - 2(x - 7)(y + 5) + 4(y + 5)^2]$$
$$-\infty < x, y < \infty$$

       Determine the parameters. Also find the value of k.

<div align="center">OR</div>

Q4 a) Define hyper geometric distribution. Derive its probability mass 8
        function stating conditions for the existence of distribution.

    b) Check whether for the sequence of random variable $\{X_k\}$, WLLN holds 6
        good or not?

        Prob. $\{ X_k = 1/\sqrt{k}\} = 2/3$ and Prob. $\{ X_k = -1/\sqrt{k}\} = 1/3$.

Q5 a) Derive moment generating function of bivariate normal distribution. 7

    b) For a bi-variate normal distribution, derive an expression for the 7
        conditional distribution of X given Y = y.

<div align="center">OR</div>

Q5 a) For bi-variate normal distribution, show that X and Y are independent, 8
        iff $\rho = 0$.

    b) If [X, Y] is Bivariate Normal with parameters-[3, 1, 16, 25, 3/5], find 6
        i.   P[3<Y<8/x=7]
        ii. P[-3 < X<3/y=-4]